ON THE SIZE DISTRIBUTION OF ESTABLISHMENTS OF LARGE ENTERPRISES: AN ANALYSIS FOR UK MANUFACTURING

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Abstract

A parsimonious explanation for the positive skew of enterprise size distributions is Gibrat's law. In large multi-establishment enterprises, do establishment-specific idiosyncratic shocks penetrate the veil of common ownership and lead to a similar regularity in the intra-enterprise, establishment size distribution? We use a maximum entropy method to derive statistically a null hypothesis on the nature of this distribution for UK manufacturing industries. We show that the degree of concentration of the intra-enterprise establishment size distribution determines the variance of the distribution of growth rates of the enterprise - and thus the risk it faces.

Key Words: Maximum Entropy, Size Distribution, Establishment, Enterprise.

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1 Introduction

For most of this century, British industry was marked by a tendency towards concentration and towards large enterprises. While there has been a pronounced decline in the employment significance of large enterprises in manufacturing since the beginning of the 1970s, they remain important. In 1991, the largest 100 employers (enterprises) in manufacturing accounted for 42 percent of total United Kingdom manufacturing output and 37 percent of total UK manufacturing employment. They employed, between them, 1.45 million persons in their 2423 *establishments*. The Standard Industrial Classification defines an establishment as:

... the smallest unit that can provide information normally required for an economic census ... employment, expenses, turnover, capital formation. Usually the principal activities carried on in an establishment fall within a single heading of the classification (steel making, sugar refining). Typically the establishment embraces all the activities carried on at a single address...

In economic terms as well as for the purposes of statistical enquiries directed at enterprises, the establishment is the smallest, "whole", economic and accounting entity.[1] Establishments span a wide range of sizes, and there are very large establishments in operation. For instance, in 1991 there were 11 establishments that individually employed over 10,000 persons.

Do the establishments operated by the large (multi-establishment) enterprises tend to be large? This issue has almost always been discussed in terms of the average size of owned establishments (Nelson, 1963; Pryor, 1972; Scherer *et. al.*, 1975; Prais, 1976; Miller, 1978).[2] The size distribution of establishments owned by large enterprises has been neglected. However, Prais (1976), investigating whether establishment sizes contributed to enterprise level concentration in UK (were large establishments responsible for the concentration in enterprise sizes ?) concluded:

... it is far from the truth to typify the giant enterprise of today as consisting of a few giant plants administered from a central office; one should rather think of the typical giant enterprise as owning a host of plants, most of which are only of medium or small size, though in some cases there may be a nucleus of large plants ... the general impression given by the figures is that the typical giant enterprise has progressively become one that owns an ever greater multitude of plants and, far from specialising in the ownership of large plants, the average size of its plants has tended to fall...

For want of data, the measures of multi-establishment operation that have been defined and used in the literature have no distributional dimension. The most commonly used measure is the 'count' of all establishments operated by the leading enterprises, regardless of size. This paper is concerned with the detection of an empirical regularity *within* the large enterprise. We are concerned with the size distribution of establishments in multi-establishment enterprises. The minimal purpose of this paper is to generate a null hypothesis about this distribution.[3] The broader objective is to highlight a fruitful direction in which further analysis of growth and of risk faced by enterprises may be undertaken.

2 Literature

In the literature on corporate growth, establishments are recognised to be distinct from enterprises only in passing. The growth of both establishments and enterprises is treated in the same way, neglecting the obvious fact that enterprises are constituted out of establishments.[4] It is an empirical matter whether multi-site, multi-activity - enterprises have establishments which evolve in a highly correlated way or somewhat independently of each other.[5]

It is well known that the striking empirical regularity of positive skew in size distributions has a parsimonious explanation in terms of a process that subjects a large number of units to identically and independently distributed multiplicative shocks - Gibrat's law. If one expects the establishments of a large enterprise to evolve somewhat independently of each other, it is reasonable to ask whether the idiosyncratic shocks that penetrate the moderating influence of common ownership and control result in some empirical regularity in the intra-enterprise, establishment size distribution. If large enterprises have a large number of relatively autonomous constituent establishments that operate in dissimilar environments, one might examine the tendency of the intra-enterprise size distribution to be characterised by positive skew.

2.1 Models of Enterprise Growth

Recent theories of competitive equilibrium in industry evolution have traced the dynamics of size - as well as of other enterprise level variables - to the stochastic process that is assumed for the evolution of variable(s) relevant to the decision problem of optimising enterprises. They emphasise learning and selection in the making of irreversible investments in an uncertain environment.[6] These models are the successors of a robust vintage of theoretical models of growth and size distributions which addressed the fact that despite a high degree of unpredictability in the motion of the individual enterprise sizes, the size distributions of enterprises were relatively stable. In particular, they were positively skewed.[7] Since this feature is seen in many economic magnitudes, these early models were designed to be context-free and are somewhat lacking in behavioural interpretation.

They explained the empirical regularity in terms of a stochastic equilibrium. The best known variants[8] assumed that enterprise size was a random variable, and defined a stochastic process directly for size over time. The random process was usually modelled as additive random shocks to the logarithm of the size - changing size by randomly distributed proportions - and was called the law of proportionate effect (LPE) or Gibrat's law. If the conditions of the central limit theorem are satisfied, the logarithm of enterprise size will follow a normal distribution.[9] Simon and Ijiri (1977) showed that significantly weaker variants of the LPE generate distributions that share the feature of positive skew.[10]

2.2 Empirical Studies

These models have been applied to both establishments and to enterprises identically, as well as independently. Virtually no direct attention - either theoretical or empirical - has been paid to establishment dynamics within the enterprise. But recent empirical work on the dynamic behaviour of establishment sizes in manufacturing suggests that significant stochastic processes *may* be at work *within* enterprises. Even though this work does not condition establishment dynamics on the owning enterprise, the results are suggestive. There is a very high degree of dispersion in establishment level growth rates even within narrow sectors - even in a well-defined sector, some establishments grow at high positive rates while some are characterised by negative growth rates (Dunne and Hughes, 1994; Dunne, Roberts and Samuelson, 1989). A significant part of the sectoral size increases (losses) are due to establishment births (deaths) and sharp size increases (drops) of existing establishments (Cabellero, Engel and Haltiwanger, 1994). Though there is significant persistence of the gains and losses - most of the size gain survives and lost size remains lost after one year, there are also conspicuous short term, transitory, gains and losses of size. Further and most important, idiosyncratic effects, those establishment-specific growth rates - as distinct from growth due to aggregate shocks and sectoral shocks - play the major role in gross sectoral size gains and losses (Davis and Haltiwanger (1989)). Hamermesh (1987) found corroboration of the above within the enterprise. Examining the output and employment data on seven manufacturing establishments of a durable goods manufacturer in the US he found that even major adjustments are establishment specific; there is little contribution from enterprise-wide demand shocks or common unobserved factors.[11]

If it were indeed true that the establishment sizes within the enterprise were driven substantially by idiosyncratic shocks, then by not properly recognising the fact that enterprises operate establishments, we miss out the constraints that probability theory sets on the joint behaviour of many units whose growth is not perfectly correlated. These restrictions apply to the distribution of growth rates of the enterprise.

By the limit argument, probabilities at the level of the individual - over time - are analogous to the cross sectional distribution. That argument has been applied to the population of all establishments, and of all enterprises. If, across the establishments operated by an individual large enterprise, the establishment growth rates were drawn independently and identically from the same distribution, then the realised cross sectional intra-enterprise establishment size distribution will converge to the probability distribution that describes the possible positions of an individual establishment within the enterprise. Combined with assumptions leading to the ergodic nature of the random process, the result is a steady state distribution. [12] [13]

What can we infer about this intra-enterprise establishment size distribution when the data is limited - as for instance, is the case with the data reported by the UK Central Statistical Office. For the class of the largest - in terms of employment as well as in terms of net output - 100 manufacturing enterprises in the UK, CSO reports only the number of businesses/establishments operated by these enterprises, their total employment, as well as their total net output. These figures yield us the mean size of the establishments operated by a representative of the class of largest manufacturing enterprises. This is the only empirical information we can depend on, while attempting to determine the shape of the intra-enterprise establishment size distribution. In the next section we proceed to do that.

3 Methodology

3.1 Principle of Maximum Entropy

Maximum entropy is a powerful and general technique of data analysis. It is applicable whenever one wants to estimate a unique vector of proportions $\mathbf{p} = (p_1 , p_2 ... p_m)$ from significantly incomplete data which could be fitted with many different vectors. Maximum entropy selects the result, containing the bare minimum of structure needed to fit the data. The principle is to maximise entropy subject to empirically known constraints.

The basic approach is simple. Let s, the establishment size operated by the large enterprise, be a random variable with an unknown probability density function F(s), about which the best *a priori* guess we can make is $F_0(s)$. Specifically, suppose we knew that the large enterprise operated establishments ranging in size from s_{min} to s_{max}. The mid-points of size classes into which establishment sizes fall are, say, s_1 , s_2 ... s_m. Restricting attention to these sizes, consider the probability distribution $(p_1 , p_2 ... p_m)$ where p_k is the probability that an establishment of size s_k is operated. We have no information on which, if any, of these sizes are favoured by the enterprise. All we know is that the favoured size, if any, lies in the set $\{s_1 , s_2 \dots s_m\}$. Thus, *a priori*, by the principle of insufficient reason we can only have recourse to the uniform distribution to describe our knowledge of establishment sizes of large enterprises ($p_1 = p_2 = \dots = p_m$).

Suppose we also do have an additional bit of information - the mean establishment size, $E(s) = \Sigma_k s_k p_k$. How should we modify our guess about F(s)? The principle of maximum entropy applies to this situation and as a criterion of choice it recommends that from the set of all probability distributions compatible with the mean value, we choose the one that maximises entropy. The entropy $H(\bar{p})$ attached to the finite probability distribution $\bar{p}$ is the number

$$H(p_1 , p_2 \dots p_n) = -\sum_{i=1}^{n} p_i \ln p_i \tag{1}$$

The entropy function has useful properties entitling it to be considered a good measure of the amount of information contained in a probability distribution. The maximum entropy probability distribution will ignore no possibility, being the most uniform subject to the constraint (Guiasu and Shenitzer, 1985; Skilling, 1984).

Note in particular that, in the language of industrial concentration, this procedure will determine the distribution with minimum concentration, given the constraints (See Theil (1967) for an early application of entropy to measure industrial concentration). Any asymmetry (or concentration) in this 'most uniform' distribution will be due to the empirically known constraint and thus reflect the asymmetry of the actual distribution. If the asymmetry in the maximally uniform distribution is marked, we must conclude that the information embodied in the data on the average establishment size reflects the underlying asymmetry, or concentration.

3.2 Maximising Entropy Subject to Constraint

We now apply the principle of maximum entropy to the problem at hand. We know the mean value, E(s) of the random variable - the mean size of establishments that belong to the large enterprise - and we have fixed the sizes that the establishments can take, $\{s_1, s_2 \ldots s_m\}$. We need the unique probability distribution:

(2)
$$p = (p_1, p_2 \ldots p_m) \; ; \; p_k \geq 0 \; ; \; \sum_{k=1}^{m} p_k = 1$$

satisfying the additional constraint

(3)
$$E(s) = \sum_{k=1}^{m} s_k \, p_k$$

such that entropy $H(\bar{p})$ is maximised. Since the $H(\bar{p})$ is a concave and continuous function defined in the convex domain characterised by the constraints, and there is only one global maximum point belonging to the open set

(4)
$$p = \{ (p_1, p_2 \ldots p_m) \mid p_k \geq 0 \, , \, (k = 1, 2 \ldots m) \; ; \; \sum_{k=1}^{m} p_k = 1 \; ; \; \sum_{k=1}^{m} s_k \, p_k = E(s) \}$$

The first order conditions of the straightforward convex constrained optimisation problem yields:

(5)
$$p_k = (e^{\beta s_k}) \, / \, (\sum_{r=1}^{m} e^{\beta s_k})$$

where β is the solution of the equation

(6)
$$\sum_{k=1}^{m} (s_k - E(s)) \, e^{\beta (s_k - E(s))} = 0$$

A solution exists, and is unique if the random variable takes on at least two different values. The function is strictly decreasing between limiting values of negative and positive infinity. The solution of the exponential equation might be a transcendental number, but we can approximate β with sufficient accuracy. We can thus find the most uniform probability distribution compatible with the given mean value. When there is no constraint the solution of the maximum entropy problem is the uniform distribution. The substance of the question is: how much bite does the constraint - average establishment size - have?

$$\sum_{k=1}^{m} (s_k - E(s)) \, e^{\beta (s_k - E(s))}$$

4 Data and Results

Using data reported in the Reports on United Kingdom Annual Census of Production (ACOP), for each year, we have worked with 'representative large enterprises'. For example, in 1991, the largest 100 enterprises employed 1.45 million persons in 2423 establishments. We restrict attention to the case of an average large enterprise employing 13248 persons in 24 establishments. Our aim is to find the probability density function that characterises the size distribution of these 24 establishments.

To proceed with the analysis one must make some assumptions about the range of establishment sizes operated by the class of largest enterprises - about the support of the probability distribution. In most cases, we can approximate the upper bound, s_m.[14] The 'largest' admissible size in the intra-enterprise establishment size distribution - we determine as the average size of the largest 100 establishments, interpolated from the size distribution of establishments reported by ACOP.

What of the smallest establishment operated by the largest enterprises? We know that these large enterprises must have operated establishments of a lesser size than the reported average size of their establishments. For instance, in terms of employment size, in 1991, the largest 100 enterprises must have operated establishments employing less than 547 persons. So we place s_1 below this average. In the reported results, we have placed the lowest sizes in terms of employment, at 50 workers; and in terms of net output, at half a million pounds.[15] In determining the most symmetric probability density function that characterises the distribution of (the 24, in employment terms, in 1991) establishments, we restrict attention to a set of eleven employment sizes that span from the lowest admissible size to the largest admissible size in geometric progression.[16]

We implemented the Newton Raphson algorithm in GAUSS to solve for β and computed the maximum entropy probability distributions relating to the 100 largest UK enterprises classified to manufacturing. We considered size both in terms of total employment and in terms of net output. We computed the distributions for *all manufacturing* for each year between 1987 and 1991, as well as for all 15 SIC 2 digit industry classes for 1991.

Figures 1, 2, 3 and 4 present the maximum entropy distributions for 1991 and 1990, fitted as described, with establishment size presented in logarithms. These figures give us a visual impression of the nature of the distribution.

It is useful to supplement the figures with the usual measures that summarise the distribution function. Tables I and II present the distributional statistics - measures of central tendency, dispersion, shape, and concentration - of the maximum entropy probability distributions.

It is important to remember that these probability distribution functions are statistically generated null hypotheses about the actual distributions. They are the most uniform probability distribution functions that are consistent with what is known of the data and thus are not estimates of the actual distributions. Thus they are not suitable for further analysis in terms of comparisons across years. However, it is evident that across the years, even the most uniform, minimally concentrated, intra-enterprise establishment size distribution was characterised by asymmetry in terms of a positive skew. This is evident both in terms of employment size and in terms of net output size. Thus we may conclude that the information contained in the average establishment size reflects a significant underlying asymmetry and concentration of the actual intra-enterprise establishment size distribution.

The two digit industry analysis is presented in Table III and IV.

The following observations might be made on the disaggregated analysis. While for "all manufacturing", the UK Central Statistical Office (CSO) reports statistics for the largest 100 enterprises, for the SIC 2 digit industry classes, CSO aggregates a variable number of enterprises into the largest class for which data is furnished. This preserves the confidentiality of details of individual undertakings. These (variable) numbers - of enterprises aggregated into the "large" class at individual industry level - tend to be high relative to the total sizes of the industries, when compared to 100 enterprises from all manufacturing. Thus generally, at the industry level, the class of large enterprises would tend to reach deeper and include the smaller and less diversified enterprises. To that extent, a higher degree of correlation in establishment growth, and the above reported consequence, are not surprising.

Industries vary considerably in the number of establishments that their largest enterprises operate. The largest enterprises in industries such as Instrument Engineering (37), Other Transport Equipment (36) and Other Manufacturing Industries (49) are single establishment enterprises. Technology could lead the largest enterprises in some industries to operate fewer establishments than comparable enterprises in other industries. This has two implications for differences across individual industries. First, the degree of diversification of large enterprises can be specific to their primary activity. Some industries are likely to be marked by a higher degree of correlation in establishment growth. Second, industry dynamics characterised by Gibrat's law produce positively skewed steady state distributions in the limit, as both time and the number of units tend to infinity. The smaller the number of establishments operated by the large enterprise, the less likely is the steady state to be observed.

5 Variance of the Distribution of Growth Rates of a Multi-Establishment Enterprise

Now we turn briefly to the significance of this size distribution. We consider here the simplest of models - an enterprise whose size is the sum of the sizes of its establishments. If S_t is the size of the enterprise, and s_{jt}, the size of the *j*th establishment operated by the enterprise,

(8) $$S_t = \sum_{j=1}^{n} s_{jt}$$

The rate of growth of the enterprise is where the *j*th establishment's size share θ_{jt} weights its growth rate.

(9) $$\dot{S}_t = \sum_{j=1}^{n} \theta_{jt} \, \dot{s}_{jt}$$

It is elementary that if the growth rates of the establishments are drawn independently and identically from the same distribution, the variance of the growth rate density of the enterprise must fall with some increasing function of n. Hymer and Pashigian (1962), and Prais (1976) in their discussion of this issue reasoned in terms of the arithmetic mean of the growth rates of the equal sized constituent units. Relaxing that construction, the variance of the growth rate of the enterprise is

(10) $$V(\dot{S}_t) = \sum_{j=1}^{n} \theta_{jt}^2 \, V(\dot{s}_{jt}) + \sum_{j=1}^{n} \sum_{i=1, i \neq j}^{n} \theta_{jt} \, \theta_{it} \, Cov(\dot{s}_{jt}, \dot{s}_{it})$$

The covariance term recognises the common shocks. If the growth rates of the individual establishments are assumed to be drawn from the same distribution, if r is the common correlation coefficient across all pairs of enterprises, j and i where $i \neq j$, then:

(11) $$V(\dot{S}_t) = \sum_{j=1}^{n} \theta_{jt}^2 \, V(\dot{s}_{jt}) + \sum_{j=1}^{n} \sum_{i=1, i \neq j}^{n} \theta_{jt} \, \theta_{it} \, V(\dot{s}) \, r$$

Then the relationship between the variances of the two growth rate distributions, of the enterprises and of the establishments is

(12) $$V(\dot{S}_t) = V(\dot{s}) \left(\sum_{j=1}^{n} \theta_{jt}^2 + 1 \right) (r/2)$$

Note that $\Sigma_{j=1}^{n} \theta_{jt}^2$ is a distributional measure, here referring to the size distribution of establishments operated by the enterprise. It is the well known Hirschman-Herfindahl measure of concentration of a distribution of sizes.[17] Thus this distributional feature determines the variance of the distribution from which the growth rate of the enterprise is drawn. The less concentrated the intra-enterprise establishment size distribution, the less is the variance of the distribution of growth rates of the enterprise.

Concentration and the variance of the growth rate move together. *Ceteris paribus*, a tendency towards equal sizes of establishments decreases concentration. The minimum value is achieved when the shares are equal. When the enterprise sets up and closes down establishments, the distribution grows less concentrated if there is 'entry' with shares below some level s, and similarly if the 'exit' is of an establishment with a share above that level. Exit of low share establishments increases concentration. The concentration index ranges in value between 1/n and 1. The lower limit, when all sizes are equal, falls with 1/n. Variance also falls with 1/n if the establishment size distribution within the enterprise has stable moments since

$$(13) \qquad \sum_{j=1}^{n} \theta_{jt}^{2} = E(s_t) \; / \; (n \; V(s_t))$$

where the numerator is the mean establishment size and the denominator is the variance of the size distribution of establishments operated by the enterprise.

Thus the growth rate distribution of an enterprise with a less concentrated distribution of establishment sizes stochastically dominates the growth rate distribution of an enterprise with a more concentrated distribution of establishment sizes.

It has been found empirically that the variance of growth rates falls slightly with the size of enterprises.(Dunne and Hughes, 1994; Geroski and Machin, 1992). This attenuation of the fall in variance has been attributed to the possibly high correlation between establishment growth rates (See for instance, Prais, 1976). This suggestion is reasoned in terms of an enterprise consisting of independently evolving equal sized constituents. Then the variance ought to fall with the number of establishments - and this number is generally correlated with size and age.

The above analysis suggests another explanation. The increase in the number of establishments operated by growing enterprises might be offset somewhat by a higher degree of concentration of the size distribution of their constituents. Note that the variance of growth rates responds more to the distributional feature than to the correlation coefficient.

6 Summary

It is widely recognised that over the last decade large enterprises have tended to reorganise themselves around their core capabilities. There has been a move away from conglomerate type diversification. Enterprises have tended to deploy their core competencies towards long run competitive advantage, and in the intensively competitive climate, competitive advantage has largely been based on new product and process development projects. Spinoffs and subsidiaries that deliberately separate core competencies have helped organizations to be competitive through flexibility. Diversification is increasingly through developing collections of business units founded on core capabilities. These have been called 'network' organizations (Hinterhuber and Levin, 1994).

Enterprises enter, grow, decline or exit through setting up, buying out, expanding, contracting, closing down or selling their establishments. Large enterprises have tended to permit operational and financial autonomy to subsidiary establishments. To the extent that a large enterprise is a dynamic composite of relatively independent subsidiary establishments, the size distribution of its establishments might be distinctively asymmetric and comparatively stable. Limitations of data have in the past prevented any detailed study of this size distribution. In this paper we have attempted to use the limited available information to derive statistically, a null hypothesis on the nature of this size distribution. We did this by determining the maximally uniform distribution subject to the constraints known from the limited data. We found this distribution to be characterised by a pronounced positive skew, which must reflect the underlying asymmetry.

We also considered the significance of this - intra-enterprise establishment - size distribution in determining the variance of the distribution of growth rates of the large enterprise itself. It is clear that the distribution of growth rates of an enterprise with a less concentrated distribution of establishment sizes, stochastically dominates the distribution of growth rates of an enterprise with a more concentrated distribution of establishment sizes. Recessions might have a cleansing effect (Caballero, 1994) and that effect could reach into the interior of the enterprise, yet the closing down of small establishments will increase the concentration of the establishment size distribution and thereby increase the variance of distribution from which the enterprise growth rate is drawn. This translates into higher risk faced by the enterprise, and so the analysis above has implications for the analysis of enterprise behaviour in recessions.

Notes

1. Since 1987 the 'company' has been the reporting unit for the United Kingdom Annual Census of Production. But for large mixed activity companies, separate returns are required for each production activity on an *establishment* basis. The mixture of reporting units are called businesses by the Central Statistical Office (CSO). For most businesses, the returned data is appropriate to a single activity heading of SIC(80) falling within a single geographical region. An *enterprise* is one or more businesses [establishments] under common ownership and control. Introductory notes to the yearly Report on the Annual Census of Production published by the CSO provide definitions.

2. Empirically, the relationship between the average size of the owned establishment and the size of the owning firm is robust - the larger the firm, the larger the average size of establishment.

3. The logical status of a null hypothesis is the same as that of a theory in that the null hypothesis and the theory are conjectured to be approximately valid and to be worth testing. Usually null hypotheses are assumed to be formulated before statistical theory is invoked in testing them. Statistical theory is in itself weak in suggesting or generating null hypotheses. In the problem at hand, statistical theory can be used rather effectively in formulating the null hypothesis.

4. A notable exception - though his central concern there is not corporate growth - is Lyons (1980), where a measure of minimum efficient plant size is determined from a model of firm growth through plant starts.

5. See Marginson, *et. al.* (1988), for studies conducted from an industrial relations perspective, providing evidence on the autonomy of constituent establishments of large firms.

6. See Dixit and Pindyck (1994) for a review. For competitive equilibrium models of industry evolution, see Jovanovic, 1982; Ericson and Pakes, 1989; and Hopenhayn, 1991.

7. The empirical research into the goodness of fit of theoretical distributions to corporate sizes have not been very successful in distinguishing the nature of establishment size distributions from firm size distributions. The tests of the goodness of fit of candidate theoretical distributions are of low power in discriminating between very similar distributions (Ijiri and Simon, 1977). However, the size distributions of both establishments and firms are significantly positively skewed (Simon and Bonini, 1958; Hart and Prais, 1956; Steindl, 1965; Quandt, 1966; Clarke, 1979). And for firms as well as for establishments in the upper ranges, the Pareto distribution fits better (Engwell, 1973; Steindl, 1965; Clarke, 1979). There is a robust tendency for the upper range of the distributions functions to bend down, a tendency to concavity (Simon and Bonini, 1958; Steindl, 1965). The stability of the distribution is only moderate. There have been prolonged stints of increasing and then decreasing variances, and a 'stable' (equilibrium) size distribution is not evident.

8. There are varieties of models according to whether size and time are considered discrete or continuous. See Champernowne, 1953; Kalecki, 1945; Simon, 1955; and Wold and Whittle, 1957.

9. Since this process is a model of diffusion if it is unrestricted, various types of restrictions were employed to stabilise the process and obtain a time-invariant distribution of the size of this firm. Under additional assumptions that individual sizes evolve according to identical processes and independently of each other, the steady state distribution of the representative firm size is the same as the steady state cross sectional size distribution of firms.

10. Such as serial correlation in the size changes of individual firms, and substantial variation in the expected growth rates of individual firms, restricted only by the expected growth rate of firms in each size class being markovian. Simon (1955) showed that when incorporating the LPE in the transition matrix of a wide range of stochastic processes, a range of reasonable assumptions can lead to Log normal, or Pareto, or Yule, or Fisher's log distribution as the resulting steady state distribution, and all of these are highly skewed and bear significant

resemblance to those empirically observed. The steady states are very different if the LPE were not a feature.

11. It is appropriate to situate these findings in the context of the wider branch of empirical research that examine the validity of the assumptions on the growth rates of firms in the theoretical models described above. The balance of evidence is that for firms as well as for establishments (i) Growth rates tend to decline with size (ii) Variance of the growth rate distribution tends to fall with size (iii) Across all sizes the growth behaviour is noticeably but not strongly inconsistent with the postulate that growth rate is independent of size - there is some evidence of positive serial correlation in growth - but the LPE postulate does hold better for large firms than for the small firms. Some important early studies were Rowthorn, 1971; Singh and Whittington, 1968; Hymer and Pashigian, 1962; and Mansfield, 1962. Recent studies include Leonard, 1986; Hall, 1987; Evans, 1987; Geroski and Machin, 1992; and the references therein.

Another branch of empirical research focused on the birth and death of firms as well as of establishments. The main findings are, for firms as well as for establishments (a) Probability of failure is negative correlated with size and with age. (b) Multi-establishment firms face lower probabilities of failure and so do establishments operated by multi-establishment firms. (c) Hazard rates are lower in industries where product differentiation is easier, as well as in industries that experience faster rates of growth. See Dunne and Hughes, 1994; Audretsch, 1991; Audretsch and Mahmood, 1991; Mata and Portugal, 1992; and references.

12. If the establishment growth rates are independent of size, then we might expect a positively skewed distribution of these establishment sizes.

13. Not all shocks are idiosyncratic. They have different origins. There are aggregate shocks, sectoral shocks, enterprise level shocks and finally the idiosyncratic shocks. As these shocks cascade on to the establishments, there is likely to be some correlation between the realised growth rates of the constituent

establishments. When there is less than perfect correlation, the relation between the probability statements for the individual establishment, and the behaviour of the cross sectional distribution of establishments will agree, but both will be conditioned on the entire path of the aggregate shocks. If the path of the aggregate shock is not constant, then the cross sectional distribution may not converge to a stationary state.

14. Enterprises must be at least as large as the establishments they operate. On average, the largest establishments must be operated by the largest enterprises. For example, from the 1991 data on the employment size distribution of establishments, by interpolation, we find that the largest 100 establishments had an average employment of 5891 persons (admittedly, ranging from a minimum size of about 2850 persons). At the same time, the largest 100 enterprises fell in the size range defined as employing at least 5000 persons.

15. We have carried out sensitivity analyses to examine the consequences of allowing different ranges. The qualitative conclusion remains unchanged. In results we report here, we are being conservative in working with a limit well below the average size. The closer the lower limit to the average, the more positively skewed is the "most uniform" distribution.

16. Again, we have carried out sensitivity analyses and have concluded that different size vectors do not make any material difference to the results.

17. As a general measure of concentration, this index is used with the shares raised to some positive power. While powers less than one focus on the distribution of the smaller units, a power of two focuses on the distribution of the larger units.

FIGURES AND TABLES

Table I. Maximum Entropy *Employment* Size Distribution of Establishments operated by Large Enterprises.
***All Manufacturing* -- UK.**
Distributional Statistics

Year	Central Tendency		Dispersion	Other Moments		Concentration	
	Mean	Median	Coeff. of Variation	Skew-ness	Kurt-osis	H-H Index	Nos. Equiv't
1991	547	209	150.09	3.24	16.44	0.134	8
1990	550	213	151.50	3.33	17.5	0.125	8
1989	495	215	149.88	3.51	20.07	0.108	9
1988	488	215	149.50	3.52	20.21	0.105	10
1987	461	218	148.26	3.61	21.73	0.102	10

Table II. Maximum Entropy *Net Output* Size Distribution of Establishments operated by Large Enterprises, *All Manufacturing* - UK. Distributional Statistics

Year	Central Tendency		Dispersion	Other Moments		Concentration	
	Mean	Median	Coeff. of Variation	Skew-ness	Kurt-osis	H-H Index	Nos. Equiv't
1991	21	6	183.11	3.15	14.04	0.188	5
1990	20	6	186.85	3.37	16.22	0.174	6
1989	17	3	189.92	3.82	21.61	0.154	7
1988	15	3	186.49	3.75	20.92	0.145	7
1987	13	3	182.88	3.82	22.06	0.137	7

Table III. Maximum Entropy *Employment* Size Distribution of Establishments operated by Large Enterprises, UK Manufacturing -1991 - Distributional Statistics

SIC Class	Central Tendency		Dispersion	Other Moments		Concentration	
	Mean	Median	Coeff. of Variation	Skew-ness	Kurt-osis	H-H Index	Nos. Equiv't
22	598	241	115.67	1.58	4.57	0.37	3
23	44	36	71.47	2.67	13.79	0.25	4
24	285	149	116.10	2.48	10.08	0.33	3
25	740	374	112.45	1.36	3.71	0.59	2
31	232	162	93.55	1.67	5.26	0.42	2
32	373	200	107.06	1.65	5.01	0.50	2
34	679	256	119.10	1.61	4.67	0.37	3
35	2357	1001	111.62	0.96	2.45	0.75	1
36	2536	1462	94.37	0.53	1.69	0.77	1
37	322	207	80.70	0.84	2.44	0.96	1
41/42	628	296	139.08	2.41	9.01	0.17	6
43	318	195	107.78	1.86	6.07	0.27	4
45	625	261	124.89	1.86	5.82	0.42	2
46	180	120	92.63	2.16	8.17	0.48	2
47	226	144	109.73	2.68	12.11	0.26	4
48	339	182	98.66	1.42	4.13	0.53	2

Notes:
(i) The support of the maximum entropy distribution for industry 23 has a lower bound of 25 employees.
(ii) The largest enterprises in industry 49 has on average fewer than 2 establishments.

Table IV. Maximum Entropy *Net Output* Size Distribution of Establishments operated by Large Enterprises, UK Manufacturing - 1991 - Distributional Statistics.

SIC Class	Central Tendency		Dispersion	Other Moments		Concentration	
	Mean	Median	Coeff. of Variation	Skew-ness	Kurt-osis	H-H Index	Nos. Equiv't
22	24	7	134.11	1.52	4.08	0.40	3
23	3	2	122.34	2.50	10.14	0.42	2
24	10	4	148.83	2.40	8.73	0.41	3
25	46	16	125.55	1.16	2.86	0.60	2
31	6	3	122.22	1.75	5.31	0.53	2
32	12	5	126.69	1.55	4.32	0.58	2
34	20	6	130.98	1.49	4.00	0.41	3
35	70	21	127.25	1.13	2.75	0.80	1
36	89	36	103.93	0.59	1.67	0.81	1
41/42	29	9	156.28	2.04	6.29	0.25	4
43	5	3	122.71	1.92	6.17	0.29	3
45	10	3	131.89	1.79	5.38	0.39	3
46	4	2	120.28	2.18	7.79	0.56	3
47	13	4	158.35	2.56	9.66	0.41	2
48	11	5	122.29	1.47	4.06	0.64	2

Notes:
(i) The largest enterprises in industries 37 and 49 has on average fewer than 2 establishments.

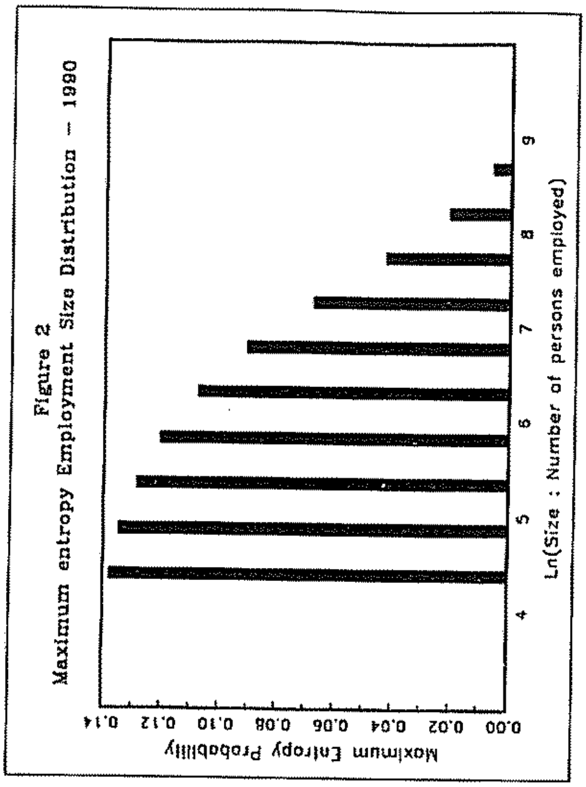

Figure 2
Maximum entropy Employment Size Distribution – 1990

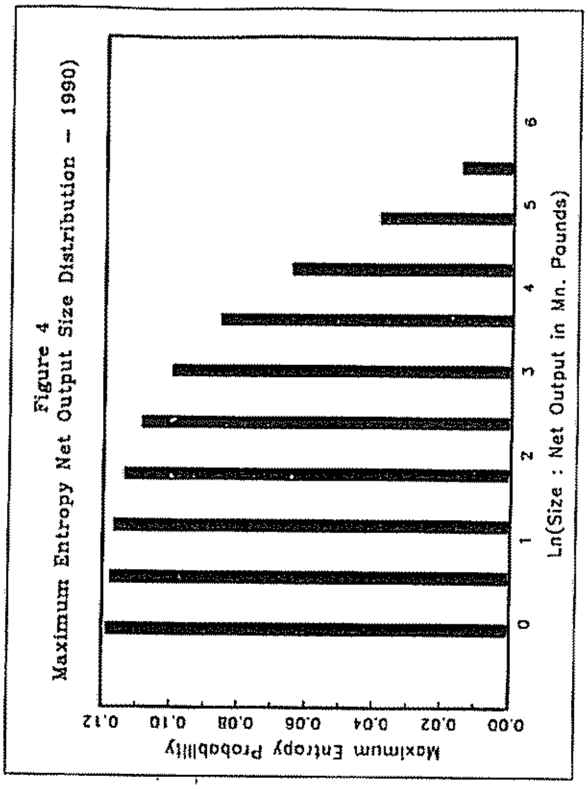

Figure 4
Maximum Entropy Net Output Size Distribution – 1990)

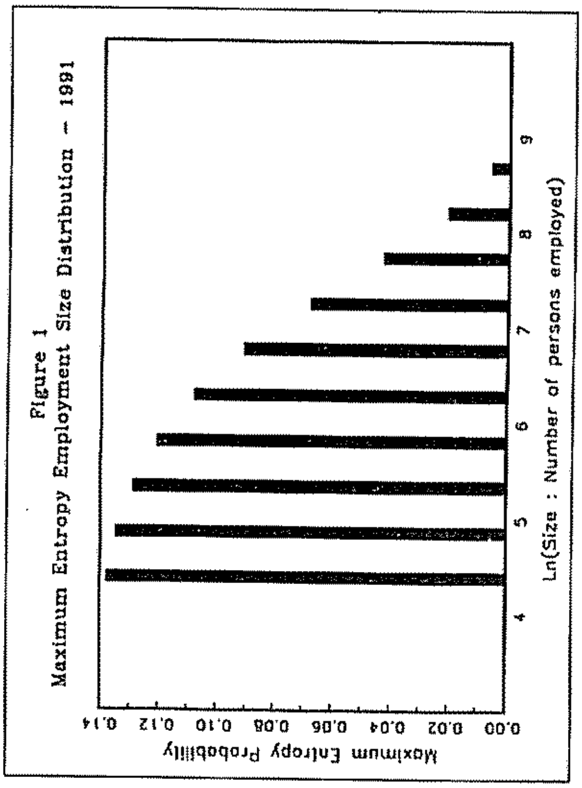

Figure 1
Maximum Entropy Employment Size Distribution – 1991

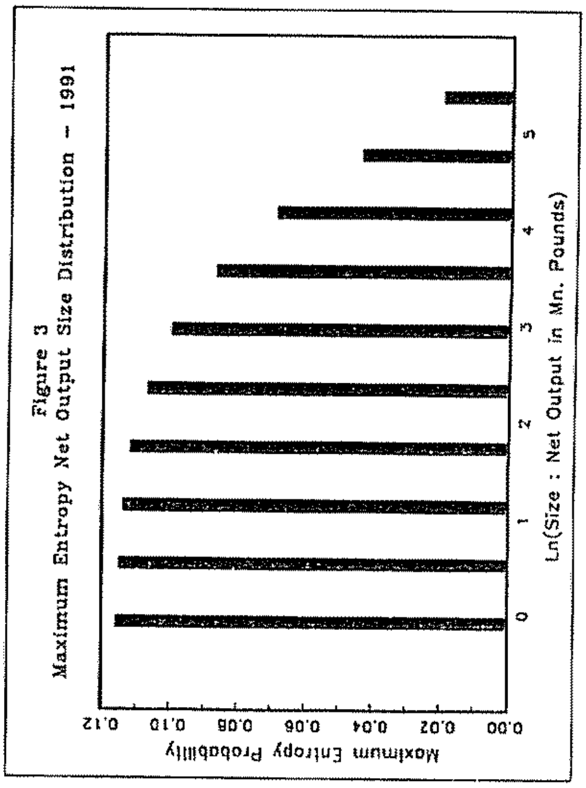

Figure 3
Maximum Entropy Net Output Size Distribution – 1991

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