A BRIEF NOTE ON SOCIAL MOBILITY AND INCOME DISTRIBUTION

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WP 538
June 2023

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Centre for Business Research, University of Cambridge
Working paper No. 538

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June 2023

Abstract

Using a simple model based on Gibrat's Law of Proportionate Effect, this note demonstrates formally how, in a dynamic setting, earnings inequality is generated. The distribution of earnings in each generation is determined by parental earnings in the previous generation and by random effects uncorrelated with parental earnings. The asymptotic distribution of earnings is log-normal. The paper concludes with a comparison of Sweden and the USA. This comparison suggests that random effects are more important than intergenerational transmission in explaining why earnings inequality is much greater in the USA than in Sweden.

JEL Codes: C20, D31, J31

Keywords: Inequality, social mobility

Acknowledgements

Thanks to Miles Corak for his data and to Bill Martin for his valuable comments. Commenting on an earlier draft, Sam Bowles drew my attention to an article which he co-authored and which employed an analytical framework similar to the one used here. (.Monique Borgerhoff Mulder et al, *Intergenerational Wealth Transmission and the Dynamics of Inequality in Small-Scale Societies* Science 326,2009)

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1. Introduction

This note was inspired by the evidence on social mobility and income distribution presented in a recent book by Aghion, Antonin and Bunel.[1] Their evidence includes a diagram, taken from an article by Corak, which shows a strong positive correlational between social mobility and inequality in advanced economies.[2]

Corak explains this correlation as follows:

> First, parents may transmit economic advantages through social connections facilitating access to jobs, admission to particular schools or colleges, or access to other sources of human capital. Second, parents may influence life chances through the genetic transmission of characteristics like innate ability, personality, and some aspects of health that are valued in the labor market. Third, parents may influence the lifetime earnings prospects of their children in subtle ways, like through a family culture and other monetary and nonmonetary investments that shape skills, aptitudes, beliefs, and behavior (p 98).

Super-imposed on these parental influences are 'random effects' that are uncorrelated with parental income. A child from a poor background may strike it lucky and earn a fortune as an investment banker or football player, even though the average child from such a background remains poor. Conversely, a child with highly paid parents may earn a pittance as an artist or author.

Using a simple model based on Gibrat's Law of Proportionate Effect, this note demonstrates formally how, in a dynamic setting, the degree of social mobility and the scale of random effects jointly determine income inequality. The distribution of income resulting from this law is log-normal, which is analytically covenient and empirically fairly accurate except for the top 1%-3% of incomes [3]. Throughout this note, the term 'income' refers to earnngs before tax and transfers.

The paper ends with a comparison of Sweden and the USA. This comparison suggests that random effects are more important than intergenerational transmission in explaining why income inequality is much greater in the USA than in Sweden. The lesser importance of random effects in Sweden may reflect differences in the occupational structure of earnings in the two countries. Occupational pay differentials in Sweden are very narrow by American standards. Thus, if the child of a medical practitioner becomes a primary school teacher in Sweden, the proportionate decline in income is much less than in the USA. Such differences have major implications for the overall distribution of income.

2. The Model

Society consists of families, each of which contains one working parent and one non-working child. The parent works for one period only. At the end of the period, this parent retires or dies, and the child becomes a parent and takes on the role of earner. Let y_t be the logarithm of parental income in period t and suppose that

$$y_t = g_t + \phi y_{t-1} \tag{1}$$

where $0 < \phi < 1$, and the g_t are stochastic, independent normal variables with the same variance v_g. Moreover, g_t is independent of y_0. The parameter ϕ measures the persistence of family income differentials across generations. Thus, $1 - \phi$ is an index of social mobility.

From equation (1) it follows that

$$y_t = \sum_{s=1}^{s=t} \phi^{t-s} g_s + \phi^t y_0`$$

The variance of y_t is

$$v_{y_t} = \sum_{s=1}^{s=t} \phi^{2(t-s)} v_g + \phi^{2t} v_{y_0} \tag{2}$$

Since $0 < \phi < 1$, the above equation can be written as follows

$$v_y^* - v_{y_t} = \phi^{2t}(v_y^* - v_{y_0}) \tag{3}$$

where

$$v_y^* = \frac{v_g}{1-\phi^2} \tag{4}$$

In the limit, as t goes to infinity, y_t converges to a normal distribution with variance v_y^*.

3. Response to parameter shifts

The index of inequality v_y^* is an increasing function of the variance of random effects (v_g) and a decreasing function of intergenerational mobility $(1-\phi)$. This is an asymptotic result. However, for values of ϕ in the range observed in the advanced economies, the variance v_{y_t} adjusts rapidly towards the new equilibrium following a one-off shift in parameters. Of the total adjustment that eventually occurs, a fraction $1-\phi^{2t}$ takes place within the first t periods. With $\phi = 0.5$ and $t = 2$ this proportion is equal to 93 percent; with $\phi = 0.75$ it is 68 percent.

Table 1 shows the trajectory of v_{y_t} for several parameter combinations. In column (1) income distribution is in steady state and v_{y_t} remains constant. Column (2) shows what happens if the variance of random effects is permanently increased at time $t = 0$. Income inequality (as measured by v_{y_t}) increases immediately from 0.267 to 0.317. In the following period inequality increases again to 0.329, by which time adjustment is almost complete. In column (3), larger random effects are offset by greater intergenerational mobility, so the distribution of income is unchanged.

4. The Gini coeffcient and international comparisons

A common measure of income inequality is the Gini coefficient. If the distribution of income is log-normal, this coefficient is a monotonic function of the variance of log income (see the appendix). Inverting this function allows us to calculate the variance of log income from knowledge of the Gini coeffcient.

Table 2 shows how to estimate the variance of random effects in two particular cases: Sweden, which has the smallest Gini coefficient of the countries in Corak's sample, and the USA which has the largest Gini coefficient . Estimation involves the following steps. First, the publishd OECD estimate of the Gini coefficent for market income (before taxes and transfers) in 2019 is used to calculate the variance of log income (v_y^*)[4]. Using this value and Corak's value for ϕ, equation (4) then provides an estimate of the variance of random effects (v_g).

The above method implies that for Sweden $v_g = 0.41$, and for the USA $v_g = 0.61$. The estimated variance of random effects is thus considerably larger in the USA than in Sweden. Table 3 explores this point further. The variance of log income in Sweden is $v_y^* = 0.44$. Suppose that the the Swedish v_g is raised to the US level, but ϕ is held constant. This will cause the Swedish v_y^* to increase from 0.44 to 0.65. Suppose, instead, that v_g is held constant, but ϕ is raised to the US level. In this case, the Swedish v_y^* increases from 0.44 to 0.52. Thus, equalising v_g has a much bigger effect on earnings inequality than equalising ϕ.

5. Conclusion

The above analysis suggests that random effects are more important than intergenerational transmission in explaining why earnings inequality is much greater in the USA than in Sweden. The low variance of random effects in Sweden is due to the structure of its labour market. Occupational pay differentials are much narrower in Sweden than in the USA, so there is less scope for large intergenerational shifts in relative earnings. For example, the child of a medical practitioner in either country may become a primary school teacher, but this will have much less effect on their earnings in Sweden than in the USA. Table 4 gives some other examples.[5].

Our estimates assume that earnings distribution is log-normal and is in steady state, but departure from these assumptions is unlikely to affect the general picture. Nor is it likely to affect the conclusion that a larger variance of random effects may be partially or completely offset by reducing the intergenerational transmission of earnings differentials.

Table 1: Trajectories of the variance v_{y_t}

	Col. (1)	Col. (2)	Col. (3)
Variance of random effects (v_g)	0.2	0.25	0.25
Intergenerational mobility ($1-\phi$)	0.5	0.5	0.75
	v_{y_t}	v_{y_t}	v_{y_t}
$t=0$	0.267	0.267	0.267
$t=1$	0.267	0.317	0.267
$t=2$	0.267	0.329	0.267
$t=3$	0.267	0.332	0.267
Limit $t=\infty$	0.267	0.333	0.267

Table 2. Derivation of v_g

	Sweden	USA
Gini coefficient	0.37	0.47
Variance of log earnings v_y^*	0.47	0.78
Intergenerational persistence ϕ	0.22	0.47
Implied variance of random effects $v_g = (1-\phi^2)v_y^*$	0.45	0.61

Table 3. Counterfactuals: Sweden and USA compared

		Implied variance v_y^*
(1)	Swedish ϕ Swedish v_g	0.44 (actual Sweden)
(2)	Swedish ϕ American v_g	0.65
(3)	American ϕ Swedish v_g	0.52
(4)	American ϕ American v_g	0.78 (actual USA)

Table 4. Pay Differentials in Sweden and USA in 2021

	Sweden	USA
All occupations	100	100
Cashier	64	46
Primary school teacher	99	115
Air traffic controller	191	219
Specialist physician	224	433 *

*includes family doctors

Appendix

The Gini coefficient is derived fom the underlying distribution of income as follows. Let $f(x)$ be the probability density function of this distribution. For $z \geq 0$ the cumulative density function is defined as follows

$$F(z) = \Pr(x \leq z) = \int_0^z f(x)dx \leq 1$$

The share of total income accruing to recipients with income $x \leq z$ is given by

$$H(z) = \frac{\int_0^z xf(x)dx}{\int_0^\infty xf(x)dx} \leq 1$$

The Lorentz curve is derived by plotting $H(z)$ against $F(z)$. The area under this curve is

$$B = \int_0^\infty H(z)\frac{dF(z)}{dz}dz = \int_0^\infty H(z)f(z)dz$$

The Gini coefficient is equal to $1 - 2B$.

The above formulae were used to write a MatLab program for calculating the Gini coefficient of an arbitrary lognormal distribution.

Notes

1 Aghion, P.,Antonin,C., and Bunel, S. (2021), *The Power of Creative Destruction*, Belknap Press, London.

2 Corak, M, 'Income inequality, Equality of Opportunity and Intergenerational Mobility', *Journal of Economic Perspectives* 27, no.3, (2013), 79-102, Fig.1.

3 Fabio Clementi & Mauro Gallegati, 2005. 'Pareto's Law of Income Distribution: Evidence for Germany, the United Kingdom, and the United States,' Microeconomics 0505006, University Library of Munich, Germany.

4 OECD data are from https://data.oecd.org/inequality/income-inequality.htm.

5 Swedish data are from https://www.scb.se/en/finding-statistics/sverige-i-siffror/salary-search/.

US data are from https://www.bls.gov/oes/current/oes_nat.htm.