Research Papers
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PRICING EQUITY DEFAULT SWAPS USING STRUCTURAL CREDIT MODELS

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WP 12/2004

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August 31, 2004

Abstract: In early 2004, new equity-credit hybrid derivatives that offered a larger spread than vanilla credit default swaps were developed. At the centre of this development was the equity default swap (EDS), which is the subject of this paper. Structural credit models allow the simultaneous modelling of a firm's credit quality and equity value, making them a natural framework to price equity-credit hybrid derivatives. A closed-form expression for the spread of an equity default swap, which incorporates the legal risk of the derivative, is derived in terms of parameters of a general structural model. A specific structural model, that developed by Leland & Toft, is calibrated by equity data and then used to investigate properties of the EDS spread. It is seen that an equity default swap with a low trigger price can have a substantially greater annual spread than a credit default swap. Also, it is shown that unless the dividend yield is very high, the EDS spread increases as a firm's debt-equity ratio increases, assuming that the firm's asset volatility is constant. However, if there are two reference firms with different debt-equity ratios but the same equity volatility, it is shown that there is a complex relationship between EDS spreads.

Keywords: equity-credit hybrid derivatives, equity default swaps, structural credit models.

1. Introduction

By the end of 2003, it was becoming increasingly difficult in many countries to structure investment-grade credit portfolios that had significant returns, e.g. Sawyer (2003) reports that the CJ50 Index, which tracks the spreads of the 50 most liquid five-year credit default swaps in Japan, fell from 80 basis points at the beginning of 2003 to only 30 basis points towards the end of 2003. In response to this, new derivatives whose value depend on both the credit quality and the equity value of the reference firm were developed. These equity-credit hybrid derivatives have a wider spread than the vanilla credit default swap; this allows institutions that can only deal in derivatives with investment-grade reference firms to trade products that have spreads similar in magnitude to those seen on credit default swaps with speculative-grade reference firms. At the centre of this new development in hybrid derivatives is the equity default swap (commonly abbreviated to EDS), which is the subject of this paper.

The buyer of an equity default swap makes a series of payments until either a payoff event occurs or the derivative expires, while the seller makes a single payment if a payoff event occurs before the expiry of the EDS. There are two possible payoff events in an equity default swap: a credit event on the reference bond (as in a credit default swap) or a fall in the price of a single share in the reference firm to a pre-defined level, which is often referred to as the trigger price. The trigger price is usually set at significantly below the equity price[1] at the start of the derivative, e.g. a trigger price of around 30% of the equity price at the beginning of the contract is relatively standard. Therefore, equity default swaps provide protection against credit events and a large fall in the equity price of the reference firm. As an example, some firms saw their equity price fall by more than 90% after the technology bubble of the late 1990s burst, but they did not default on any debt. In this situation, a payoff event would have been triggered in an EDS (unless the trigger price was set extremely low), but not in a credit default swap.

One approach to pricing equity-credit hybrid derivatives is to use a two-factor model, with one factor linked to the equity value of the reference firm and the other factor linked to the firm's credit quality; these factors would clearly have to be correlated due to the links between credit risk and equity risk that are described, for instance, in Medova & Smith

[1] Throughout this paper, equity price refers to the price of a single share in a firm.

(2004). However, this two factor approach can be simplified by using a structural credit model. In the structural approach to credit modelling, the market value of a firm's assets is usually modelled[2] and default is assumed to have occurred on all of the reference firm's outstanding debts when the process hits a default boundary. As equity can be viewed as a call on the firm's assets, there is a deterministic relation between the value of a firm's assets and the value of the firm's equity. Therefore, structural models allow the simultaneous modelling of a firm's credit quality and equity value. As a result, equity-credit hybrid derivatives can be priced using a one-factor model (the factor being the asset value of the reference firm), making structural models a natural framework to price derivatives such as equity default swaps.

An outline of the rest of this paper is as follows. In the next section, a closed-form expression will be derived for the spread of an equity default swap in terms of parameters of a general structural model. This expression will incorporate the legal risk of the derivative. In Section 3, it is explained how a particular structural model, that proposed by Leland & Toft (1996), can be calibrated using equity data. The calibrated model is then used to investigate properties of the EDS spread, before conclusions are drawn in Section 4.

2. Pricing an Equity Default Swap

In this section, a closed-form expression for the spread of an equity default swap is derived in terms of the parameters of a general structural model. A number of assumptions that are consistent with many structural models are made; these assumptions are described below.

ASSUMPTION 1: The term structure of default-free interest rates is flat and known with certainty, i.e. the time-t_0 price of a default-free bond that promises a payment of one unit at a future time t_1 is $P(t_0, t_1) = \exp[-r(t_1 - t_0)]$, where r is the (instantaneous) default-free rate of interest, which is constant over time.

[2] Some structural models, such as Goldstein, Ju & Leland (2001), model earnings before interest and tax rather than the asset value of the firm.

ASSUMPTION 2: Let $\boldsymbol{V}_t$ be the market value of a firm's total assets at time t. In the risk-neutral measure, the value of a firm's assets follows the lognormal process

$$\frac{d\boldsymbol{V}_t}{\boldsymbol{V}_t} = (r - d)dt + s\, d\boldsymbol{W}_t. \tag{1}$$

Both the asset volatility s and the fraction d of the value of the assets paid out to holders of the firm's debt and equity are taken to be constant.

ASSUMPTION 3: The principal value of the firm's outstanding debt F is constant. Further, a firm defaults on all of its outstanding debt when $\boldsymbol{V}_t$ hits a default boundary V^B, which is taken to be a fixed proportion of the principal value of the firm's debt, i.e. $V^B = bF$ for some constant b.

ASSUMPTION 4: The equity value of the firm is zero at the default boundary, i.e. equity holders do not receive a rebate upon default by the firm.

The first two assumptions are common in structural credit modelling. These two assumptions are made for instance in Black & Cox (1976), Leland (1994) and Leland & Toft (1996), and they are generalisations of assumptions that are made in Black & Scholes (1973), Brennan & Schwartz (1978) and Brockman & Turtle (2003). While the principal value of debt is often taken to be constant in structural modelling, it is less common to assume that the default boundary is fixed, although this assumption is made for example in Leland (1994), Longstaff & Schwartz (1995), Leland & Toft (1996) and Brockman & Turtle (2003).

Assumption 4 is consistent with the vast majority of structural models. An important consequence of this assumption for the pricing of equity default swaps is that the equity price of the reference firm will always hit the trigger price before, or at the same time as, a credit event on the reference bond. If the trigger price is strictly positive, the equity price will hit the trigger price before a credit event occurs, provided that the value of a firm's assets is modelled by a continuous process, as is done here[3]. In the special case where the trigger

[3] If the value of a firm's assets is modelled by a jump-diffusion process, as proposed for example in Zhou (1997), the equity price could jump from above a strictly positive trigger price to zero. In this case, the equity price would hit the trigger price at the same time as a credit event.

price is set at zero, the equity price will hit the trigger price at the same time as a credit event, so that the equity default swap is equivalent to a credit default swap.

Define a firm's distance to default, $\boldsymbol{X}_t$, to be the ratio of the market value of the firm's total assets to the default boundary, i.e.

$$\boldsymbol{X}_t = \frac{\boldsymbol{V}_t}{V^B} = \frac{\boldsymbol{V}_t}{bF}. \tag{2}$$

Therefore, default occurs if $\boldsymbol{X}_t$ hits one. As a consequence of Itô's Lemma, the distance to default satisfies the stochastic differential equation,

$$\frac{d\boldsymbol{X}_t}{\boldsymbol{X}_t} = (r-d)dt + s\, d\boldsymbol{W}_t. \tag{3}$$

Using results in Harrison (1990), it can be shown that if the firm has a distance to default of X_{t_0} at time t_0, then the risk-neutral probability that a firm defaults in the period $[t_0, t_1]$ is given by

$$Q(X_{t_0}, t_1 - t_0) = \Phi\left(\frac{-\log X_{t_0} - as^2(t_1-t_0)}{s\sqrt{t_1-t_0}}\right) + X_{t_0}^{-2a}\Phi\left(\frac{-\log X_{t_0} + as^2(t_1-t_0)}{s\sqrt{t_1-t_0}}\right) \tag{4}$$

where

$$a = \frac{r-d}{s^2} - \frac{1}{2}. \tag{5}$$

In some structural models that satisfy the four assumptions above, including Leland (1994) and Leland & Toft (1996), expressions for the value of a firm's equity were derived and, provided that all of the constant terms were known, were seen to be a function of only the market value of the firm's assets[4],

$$\boldsymbol{S}_t = S(\boldsymbol{V}_t). \tag{6}$$

Further, the equity value is a strictly monotonic increasing function of the asset value *ceteris paribus*, so that there is a bijection between the equity value and the value of a firm's assets. Let S^* be the trigger value, i.e. the market value of the reference firm's equity that triggers a payment by the EDS seller. Recall that a consequence of Assumption 4 is that a payoff event occurs if and only if the price of a single share in the reference firm hits the trigger price.

[4] If an expression for the market value of a firm's assets is derived using a partial differential equation, as in Leland (1994), this will often be an explicit assumption when deriving the PDE.

Therefore, the trigger value is found by simply multiplying together the trigger price and the number of outstanding shares in the reference firm. At this stage, an assumption about the capital structure of the firm is made.

ASSUMPTION 5: Firms have a fixed number of outstanding shares.

As a result of this assumption, the trigger value is constant over time. Then the corresponding value of the firm's assets is the unique solution to the implicit equation[5],

$$S^* = S(V^*) . \tag{7}$$

Note that a consequence of (7) is that the value of the firm's assets corresponding to the trigger value is constant over time. Analogous to the distance to default, define the distance to a payoff event $\boldsymbol{Y}_t$ to be

$$\boldsymbol{Y}_t = \frac{\boldsymbol{V}_t}{V^*} , \tag{8}$$

in which case, a payoff event occurs if $\boldsymbol{Y}_t$ hits one. Itô's Lemma shows that the distance to a payoff event $\boldsymbol{Y}_t$ satisfies

$$\frac{d\boldsymbol{Y}_t}{\boldsymbol{Y}_t} = (r - d)dt + s\, d\boldsymbol{W}_t . \tag{9}$$

Therefore, if a firm has a distance to a payoff event of Y_{t_0} at time t_0, then the risk-neutral probability that a payoff event occurs in the period $[t_0, t_1]$ is equal to $Q(Y_{t_0}, t_1 - t_0)$, where the function Q is given by (4).

Suppose that the notional value of the equity default swap is N and the equity default swap expires at time T^{EDS}. Provided that a payoff event has not occurred, it is assumed that the buyer of the EDS makes a payment of $c^{EDS}(t_{i+1} - t_i)N$ at time t_i $(i = 1,...,m)$ which provides protection for the period $[t_i, t_{i+1})$, where $t_{m+1} = T^{EDS}$. The value c^{EDS} is known as the annualised equity default swap spread. However, if a payoff event has occurred, the EDS

[5] Alternatively, if $\boldsymbol{V}_t$ represents the asset value per share of the reference firm, found by dividing the market value of the firm's assets by the number of outstanding shares, and F is the debt-per-share of the firm, then $\boldsymbol{S}_t$ as given by (6) is equal to the price of a single share in the reference firm. Then S^* represents the trigger price, and the value of V^* given by (7) is equal to the asset value per share that corresponds to the trigger price.

buyer makes no further payments. Hence, the payment made by the EDS buyer at time t_i can be written in the form

$$c^{EDS}(t_{i+1}-t_i)N1_{\{t^*>t_i\}}, \qquad (10)$$

where t^* is the time of a payoff event. The time-t value of the total payment made by the EDS buyer is therefore given by

$$\mathrm{E}\left(\sum_{i=1}^{m} e^{-r(t_i-t)}c^{EDS}(t_{i+1}-t_i)N1_{\{t^*>t_i\}}\right) = c^{EDS}N\sum_{i=1}^{m} e^{-r(t_i-t)}(t_{i+1}-t_i)[1-Q(Y_t,t_i-t)], \qquad (11)$$

where $Q(Y_t,u)$ is given by (4).

It is assumed that a payoff event can occur at any time. The payoff of a credit default swap depends upon the level of recovery of the reference bond upon default. However, if the payoff of an equity default swap is triggered by the equity value of the firm hitting the trigger value (which always occurs in the framework outlined in this paper), the payoff is usually taken to be a fixed proportion of the notional value of the EDS; the size of the payoff would often be stipulated in the contract of the derivative. Therefore, if a payoff event occurs before time T^{EDS}, the seller of the EDS is assumed to make a payment of wN, where w is a fixed value.

The legal risk of an equity default swap is modelled by allowing a period of length $\boldsymbol{s}$ between the payoff event and the payment by the EDS seller, where $\boldsymbol{s}$ is a random variable. The period between the payoff event and the payment by the derivative seller is likely to be shorter for equity default swaps than for credit default swaps, so that the legal risk of an EDS is lower than that for a CDS. This is because the payoff event of a credit default swap depends on whether there has been a default on the reference bond; since there may be disagreement between the buyer and seller of a CDS as to whether a default has occurred, there is a legal process, described in Henderson (2000), that occurs once a firm has defaulted before the seller of the CDS has to make a payment. However, a payment by the seller of an EDS will usually be triggered by the equity value of the reference firm hitting the trigger value (or equivalently, the firm's equity price hitting the trigger price). As the payment therefore depends on the movement of an observable variable, there is less likelihood of a disagreement about whether the payoff event of an EDS has occurred.

The time-t value of the payoff of the EDS is given by

$$\mathrm{E}\left(\int_{t}^{T^{EDS}} e^{-r(u+s-t)} wNq(Y_t, u-t)du \right), \tag{12}$$

where $q(Y_t, u-t) = \dfrac{\partial Q(Y_t, u-t)}{\partial u}$ is the probability density function of the first passage time of $\boldsymbol{V_t}$ to $V*$ in the risk-neutral measure. If it is assumed that the length of the period $\boldsymbol{s}$ is independent of $\boldsymbol{V_t}$, then after a change of variable, (12) can be written as

$$wN\mathrm{E}(e^{-rs})G(Y_t, T^{EDS}-t), \tag{13}$$

where

$$G(Y_t, T^{EDS}-t) \;=\; \int_{0}^{T^{EDS}-t} e^{-ru} q(Y_t, u)du \tag{14}$$

and $\mathrm{E}(e^{-rs})$ is simply the moment generating function of the random variable $\boldsymbol{s}$. By differentiating (4), it can be seen that the expression for the probability density function of the first passage time of $\boldsymbol{V_t}$ to $V*$ is given by

$$q(Y_t, u) = Y_t^{\frac{-2a}{s^2}} \frac{1}{\sqrt{2p}} \frac{\log Y_t}{s\, u^{3/2}} \exp\left[-\frac{1}{2}\left(\frac{au - \log Y_t}{s\sqrt{u}} \right)^2 \right]. \tag{15}$$

Using results from Rubenstein & Reiner (1991), it can be shown that

$$G(Y_t, T^{EDS}-t) = Y_t^{-a+b}\Phi[d_1(Y_t, T^{EDS}-t)] + Y_t^{-(a+b)}\Phi[d_2(Y_t, T^{EDS}-t)], \tag{16}$$

where

$$d_1(Y_t, u) = \frac{-\log Y_t - bs^2 u}{s\sqrt{u}}, \tag{17}$$

$$d_2(Y_t, u) = \frac{-\log Y_t + bs^2 u}{s\sqrt{u}}, \tag{18}$$

and

$$b = \frac{\sqrt{(as^2)^2 + 2rs^2}}{s^2}. \tag{19}$$

The time-t value of the EDS to the buyer of the derivative is given by the value of the payoff of the EDS minus the value of the total payment made by the EDS buyer:

$$\text{Value} = wN\mathrm{E}(e^{-rs})G(Y_t, T^{EDS}-t) - c^{EDS} N \sum_{i=1}^{m} e^{-r(t_i - t)}(t_{i+1} - t_i)[1 - Q(Y_t, t_i - t)]. \tag{20}$$

The following proposition is a simple consequence of (20).

Theorem 1

The par EDS spread c^* that makes the time-*t* value of the equity default swap equal to zero is given by

$$c^* = w\mathrm{E}(e^{-rs})\frac{G(Y_t, T^{EDS} - t)}{\sum_{i=1}^{m} e^{-r(t_i - t)}(t_{i+1} - t_i)[1 - Q(Y_t, t_i - t)]}. \tag{21}$$

Three special cases are now considered. First, recall from earlier in this section that a credit default swap can be thought of as an equity default swap with a trigger price of zero (or equivalently, a trigger value of zero). As a consequence of Assumption 4, V^* is equal to the default boundary V^B if the trigger value is zero. Therefore, by comparing (2) and (8), it can be seen that, in this case, the initial distance to a payoff event Y_t is equal to the initial distance to default X_t. Hence, by setting Y_t equal to X_t in (21), Theorem 1 gives an expression for the spread of a credit default swap.

The second special case is that of an equity default option, where the buyer of the derivative makes a single payment at time *t*, which gives protection against a credit event and a large fall in the reference firm's equity value until time T^{EDS}. The above theorem shows that the fee of the equity default option should be

$$c^*(T^{EDS} - t)N \;=\; w\mathrm{E}(e^{-rs})G(Y_t, T^{EDS} - t)N\,. \tag{22}$$

The final special case to be considered is where the equity default swap buyer makes *n* regular payments every year, and

$$t_i = t + \frac{i-1}{n}. \tag{23}$$

If it is further assumed that $T^{EDS} - t$ is an exact multiple of $1/n$, then Theorem 1 shows that the par EDS spread is given by

$$c^* = nw\mathrm{E}(e^{-rs})\frac{G(Y_t, T^{EDS} - t)}{\sum_{i=0}^{n(T^{EDS}-t)-1} e^{-ri/n}[1 - Q(Y_t, i/n)]}. \tag{24}$$

3. Properties of the Spread of an Equity Default Swap

In the previous section, it was only necessary to make general assumptions about default-free interest rates, the dynamics of the asset value of the firm, the principal value of outstanding debt and the default boundary to derive an expression for the spread of an equity default swap in terms of parameters of a structural model. However, three of the parameters (the initial value of the firm's assets, the asset volatility, and the net payout rate to security holders) in (21) are unobservable. In this section, the effect of observable variables on the EDS spread is investigated. So that this can be done, one solution is to calibrate a structural model using equity data. The equity value of a firm depends on assumptions about the capital structure of the firm, e.g. whether the firm issues finite-maturity bonds or perpetual bonds. Therefore, it is now necessary to focus on a specific structural model, and the model proposed by Leland & Toft (1996) is selected for use in this section.

The model developed by Leland & Toft (1996) satisfies the first four assumptions outlined in Section 2. Leland & Toft did not make, nor did they need to make, any assumptions about the number of outstanding shares in a firm. As a result, the fifth assumption made in Section 2 is also consistent with their model. The reference firm is assumed to continually issue coupon bonds at a constant rate; these bonds are all of the same seniority and have an initial time-to-maturity $T < \infty$. As a result, the firm issues bonds with a principal value of $(F/T)dt$ in the interval $[t, t+dt]$, where F is the principal value of all outstanding debt. All bonds are assumed to pay a continual stream of coupon payments at a rate of c, so that the firm pays out a total of $cFdt$ in coupon payments in the interval $[t, t+dt]$. However, the firm receives tax benefits on these coupons at a tax rate that will be labelled tax. Also, the firm experiences default costs of aV^B at the time of default. Finally, since the issued bonds have the same seniority, debt-holders receive the same fraction of par at the time of default, regardless of the bond's remaining maturity.

Theorem 2

Given the capital structure outlined above, the value of a firm's equity is given by

$$S_t = S(V_t) = V_t - \frac{(1-tax)cF}{r} - \left(\frac{(tax)c}{r} + ab\right)FX_t^{-(a+b)} - \left(1-\frac{c}{r}\right)F\left(\frac{1-e^{-rT}}{rT} - I(X_t,T)\right) - \left((1-a)b - \frac{c}{r}\right)FJ(X_t,T). \tag{25}$$

where the functions $I(X_t,T)$ and $J(X_t,T)$ are equal to

$$I(X_t,T) = \frac{1}{rT}\left(G(X_t,T) - e^{-rT}Q(X_t,T)\right) \tag{26}$$

and

$$J(X_t,T) = \frac{1}{bs\sqrt{T}}\left[-X_t^{-a+b}\Phi[d_1(X_t,T)]d_1(X_t,T) + X_t^{-(a+b)}\Phi[d_2(X_t,T)]d_2(X_t,T)\right], \tag{27}$$

while $d_1(X_t,T)$ and $d_2(X_t,T)$ are given by (17) and (18) respectively.

Proof

See Leland & Toft (1996). ÿ

As mentioned in Assumption 3, the default boundary is given by $V^B = bF$. In the model proposed by Leland & Toft (1996), the value of b is calculated endogenously to maximise the market value of the firm's equity; Leland & Toft show that the optimal default boundary is given by

$$\hat{b} = \frac{\left(\frac{c}{r}\right)\left(\frac{A}{rT} - B\right) - \frac{A}{rT} - \left(\frac{(tax)c}{r}\right)(a+b)}{1 + (a+b)a - (1-a)B}, \tag{28}$$

where

$$A = 2ae^{-rT}\Phi(as\sqrt{T}) - 2b\Phi(bs\sqrt{T}) - \frac{2}{s\sqrt{T}}f(bs\sqrt{T}) + \frac{2e^{-rT}}{s\sqrt{T}}f(as\sqrt{T}) + (b-a) \tag{29}$$

and

$$B = -\left(2b + \frac{2}{bs^2T}\right)\Phi(bs\sqrt{T}) - \frac{2}{s\sqrt{T}}f(bs\sqrt{T}) + (b-a) + \frac{1}{bs^2T}. \tag{30}$$

To calibrate the Leland & Toft model using equity data, three equations are needed that link the unobservable variables to equity variables, since there are three unobservable variables in the structural model. One equation is provided by (25), so two more equations are required.

An application of Itô's Lemma to (6) reveals that the equity value follows the process[6]

$$d\boldsymbol{S_t} = m_S(\boldsymbol{V_t})dt + \left(s\,\boldsymbol{V_t} \frac{\partial S_t}{\partial V_t} \right) d\boldsymbol{W_t}. \tag{31}$$

This can be written as

$$\frac{d\boldsymbol{S_t}}{\boldsymbol{S_t}} = \left(\frac{m_S(\boldsymbol{V_t})}{\boldsymbol{S_t}} \right) dt + s_t^S\, d\boldsymbol{W_t}, \tag{32}$$

so that by comparing (31) and (32), the equity volatility at time t, s_t^S, can be seen to be equal to

$$s_t^S = \frac{s\,V_t}{S_t} \frac{\partial S_t}{\partial V_t}. \tag{33}$$

The derivative in (33) can be found by simply differentiating (25).

A firm makes three sets of payments to security holders: dividend payments to equity-holders, and coupon and principal payments to debt-holders. However, the firm receives two sets of payments: the tax-sheltering value of the coupon payments made to debt-holders, and the money received from issuing new debt. Therefore, the net payout rate to security holders is taken to be

$$d\,dt = \frac{d_t^S S_t + (1-tax)cF + F/T - SB_t}{V_t} dt, \tag{34}$$

where d_t^S is the dividend yield at time t, $(F/T)dt$ is the principal payment made for bonds that were issued in the interval $[t-T, t-T+dt]$, and $SB_t dt$ is the time-t market value of the bonds that are issued in the interval $[t, t+dt]$. This was shown in Leland & Toft (1996) to be equal to

$$SB_t = \frac{cF}{rT} + \left(1 - \frac{c}{r}\right)\frac{F}{T} e^{-rT}[1 - Q(X_t, T)] + \left((1-a)\,b - \frac{c}{r}\right)\frac{F}{T} G(X_t, T). \tag{35}$$

Therefore, the right-hand side of (34) provides an estimate of the net payout rate of the firm at time t. The net payout rate d is then fixed at this value as d is assumed to be constant over time (see Assumption 2).

[6] Although Itô's Lemma can be used to derive the form of the drift term in (31), it is not necessary for the work here.

Three equations linking the unobservable variables to observable variables have now been derived. Therefore, given values for the equity value S_t, equity volatility s_t^S, and the principal value of outstanding debt F of the reference firm as well as r, d_t^S, c, T, tax and a, the initial market value of the reference firm's assets, the asset volatility and the net payout rate can be found by solving (25), (33) and (34) simultaneously. However, if the asset volatility of the reference firm is known as well as S_t, F, r, d_t^S, c, T, tax and a, the values of two unobservable variables remain to be found, so only two equations are needed. Hence in this case, the initial asset value of the firm and the net payout rate can be found by solving (25) and (34) simultaneously.

Once the structural model has been calibrated using equity data, the next stage is to calculate V^*, the value of the reference firm's assets that corresponds to the trigger value S^*, by solving the implicit equation (7). As the equity value is a monotonic increasing function of the value of a firm's assets, V^* can be found by repeated bisection. Then the distance to a payoff event can be calculated using (8), and the par annual EDS spread can be computed using the expression in Theorem 1.

The following two graphs shows the annual spread of a five-year equity default swap for various trigger values, which are shown as a proportion of the current equity value of the firm[7]. In these graphs, the default-free interest rate is taken to be 6%, while the reference firm is assumed to have a debt-equity ratio[8] of 100%, an equity volatility of 50%, a dividend yield of 2%, and a coupon rate of 7%. The firm is assumed to issue bonds with an initial time-to-maturity of ten years. The tax rate is 15%, and it is assumed that the reference firm loses 15% of its value upon default, i.e. $a = 15\%$. Further, the buyer of the EDS makes quarterly payments, w is taken to be 50% (so that the EDS seller makes a payment of $0.5N$ upon a payoff event), and the seller of the derivative makes a payment at the time of the payoff event, i.e. s is equal to zero with probability one.

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[7] If the number of outstanding shares is constant over time, a fall in the equity value of the firm by $z\%$ is equivalent to a fall in the price of a single share by $z\%$.

[8] The debt-equity ratio of a firm is defined as the principal value of its debt over the market value of its equity.

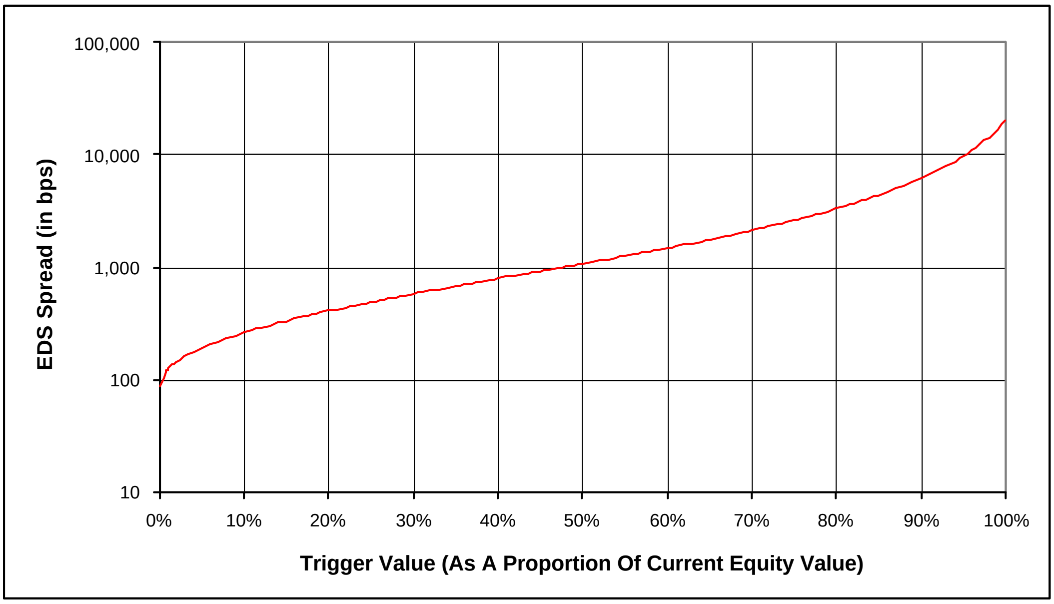

Figure 1: EDS Spreads for Different Trigger Values (Logarithmic Scale)

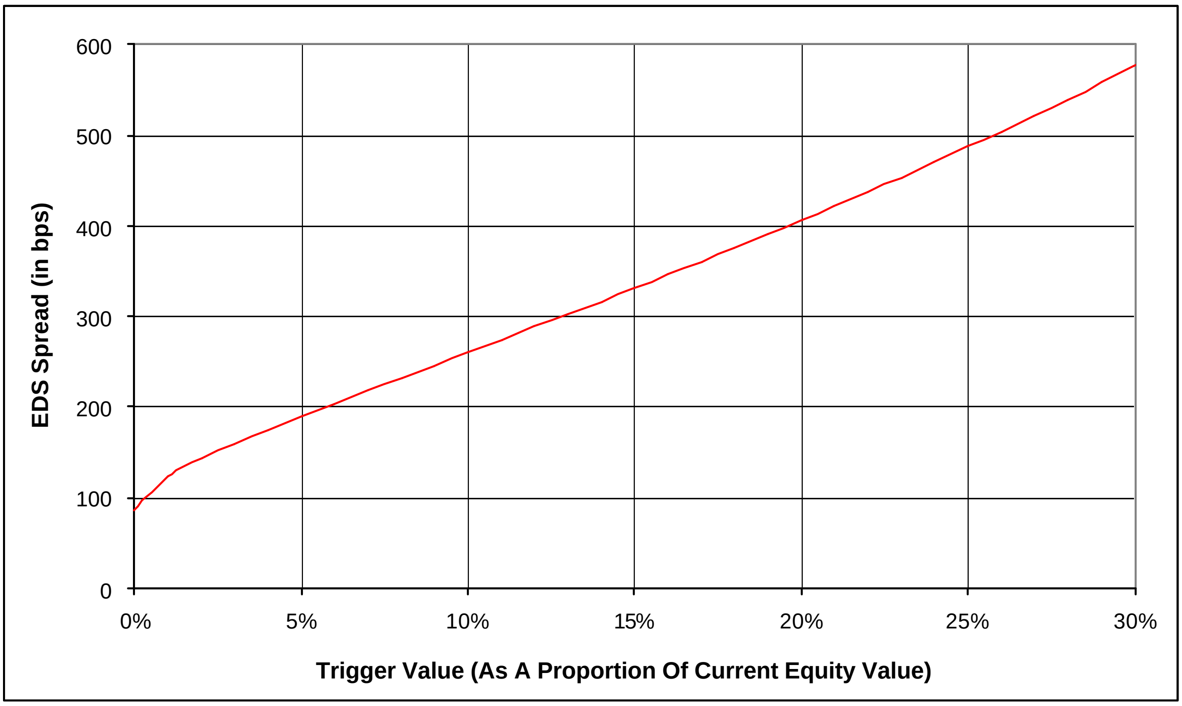

Figure 2: EDS Spreads for Different Trigger Values

These graphs show that even a very low trigger value can lead to a substantial increase in the annual spread compared with a credit default swap (equivalent to an equity default swap with a trigger value of 0). For the particular case investigated above, the EDS spread is 86.48bps if the trigger value is 0, but increases to 189.73bps and 576.39bps if the trigger value is 5%

and 30% of the current equity value respectively. Note that the CDS spread is consistent with that of an investment-grade reference firm. However, if the trigger value is 30% of the current equity value, the spread of a five-year equity default swap is similar in magnitude to that seen on credit default swaps for some speculative-grade firms.

If the trigger value is 100% of the equity value at time t, the probability of a payoff event at a time greater than t is 1. Thus, from (24), if the EDS buyer makes n payments a year, the annual EDS spread at time t is $10{,}000wn$ basis points. In Figure 1, it was assumed that $w = 50\%$ and $n = 4$, so that the EDS spread is $20{,}000\text{bps}$ if the trigger value is 100% of the current equity value.

The effect of the reference firm's debt-equity ratio on the EDS spread is now investigated. The spreads of both a one-year equity default swap (comparable to the expiry time of a reasonably long-dated equity option) and a five-year EDS (comparable to the expiry time of the most liquid credit default swaps) will be calculated, while the trigger value is set at 30% of the current equity value. Table 1 shows the EDS spreads for different debt-equity ratios while the asset volatility of the reference firm is fixed. Recall from (1) that the asset volatility of a firm is assumed to be constant over time, so that the EDS spreads in Table 1 can be thought of as the spreads of a single reference firm as its debt-equity ratio varies. It is assumed that the buyer of the EDS makes quarterly payments, and the values of r, d_t^S, c, T, w, s, tax and a used in Table 1 are the same as those used for Figures 1 and 2.

Observable Variables		Structural Variables			Equity Default Swap Spreads (in basis points)	
Initial Debt-Equity Ratio	Initial Equity Volatility	Distance to Payoff Event	Asset Volatility	Net Payout Rate	One Year EDS	Five Year EDS
25%	30.69%	2.32	25%	2.71%	3.61	126.86
50%	36.80%	1.93	25%	3.25%	44.31	267.46
100%	48.83%	1.61	25%	4.13%	301.03	538.81
200%	69.86%	1.41	25%	5.54%	1,003.91	981.17
400%	102.57%	1.29	25%	7.48%	2,113.22	1,629.34

Table 1: EDS Spreads for Different Debt-Equity Ratios While Asset Volatility is Fixed

Table 1 shows that the annual EDS spread increases as the debt-equity ratio increases, i.e. as the debt-equity of a firm increases, it is more likely to suffer a large fall in its equity value. This is due to two reasons. First, the net payout rate to security holders d is larger if the initial debt-equity ratio of the firm is greater. This can be seen by examining (34). A firm's credit quality decreases as its debt-equity ratio increases, so that the amount of money it receives from issuing new debt decreases. Therefore, unless the dividend yield is significantly greater than $(1-tax)c$, the drift term of the process modelling the asset value of the firm, given by (1), decreases as a firm's debt-equity ratio increases. This usually causes the expected growth of the firm's equity to be lower, making it more likely that the firm will suffer a large loss in its equity value. The second reason is the leverage effect, which says that a firm's equity volatility is positively related to its leverage[9]. Note from Table 1 that the Leland & Toft model is consistent with the leverage effect. Therefore, a firm would be more likely to suffer a large fall in its equity value as its debt-equity ratio increases, even if the expected growth of the firm's equity remained the same. A combination of these two factors explains the high sensitivity of the EDS spread to the firm's debt-equity ratio.

Table 2 shows the EDS spreads for different debt-equity ratios while the initial equity volatility remains fixed. This can be thought of as the EDS spreads of a cross-section of firms with the same equity volatility. Again, it is assumed that the buyer of the EDS makes quarterly payments, and the values of r, d_t^S, c, T, w, s, tax and a used in Table 2 are the same as those used for Figures 1 and 2.

Observable Variables		Structural Variables			Equity Default Swap Spreads (in basis points)	
Initial Debt-Equity Ratio	Initial Equity Volatility	Distance to Payoff Event	Asset Volatility	Net Payout Rate	One Year EDS	Five Year EDS
25%	50%	2.35	40.60%	2.82%	225.75	545.44
50%	50%	1.96	34.17%	3.41%	285.31	565.82
100%	50%	1.61	25.74%	4.20%	346.98	576.39
200%	50%	1.34	16.67%	4.89%	389.91	567.18
400%	50%	1.16	8.85%	5.32%	396.45	530.48

Table 2: EDS Spreads for Different Debt-Equity Ratios While Initial Equity Volatility is Fixed

[9] Empirical evidence of the leverage effect is provided in Black (1976), Christie (1982) and Duffee (1995).

The above table shows a complex relationship between a firm's debt-equity ratio and the spread of an equity default swap. For the particular values analysed here, the EDS spread remains positively related to the debt-equity ratio of the reference firm for equity default swaps that are reasonably close to expiry. Therefore, a highly-levered firm is more likely to suffer a 70% fall in its equity value in the next year than a firm with a low leverage, even if both firms have an initial equity volatility of 50%. However, the spread of a five-year equity default swap appears to be less sensitive to the reference firm's debt-equity ratio, and is also no longer a monotonic function of the debt-equity ratio. For the values used in Table 2, reference firms with debt-equity ratios of 50% and 200% would have similar EDS spreads, while a firm with a debt-equity ratio of 100% would have a higher EDS spread.

4. Conclusions

In this paper, a closed-form expression for the price of a new equity-credit hybrid derivative, an equity default swap, was derived in terms of parameters of a general structural credit model. The legal risk of the derivative was modelled and incorporated into the expression for the EDS spread. It was shown how a particular structural model could be calibrated using equity data, and this model was then used to investigate properties of the equity default swap spread. It is seen that an equity default swap with a low trigger value can have a substantially greater annual spread than a credit default swap. Also, it was shown that, provided that the dividend yield was not significantly larger than $(1-tax)$ times the coupon rate, the EDS spread increases as a firm's debt-equity ratio increases, assuming that the firm's asset volatility is constant. However, if there are two reference firms with different debt-equity ratios, but the same equity volatility, it was seen that there is a complex relationship between EDS spreads.

The equity-credit hybrid derivative market is already producing more complicated products, such as the equity collateralised obligation (ECO), which is an equity-credit hybrid analogy of the collateralised debt obligation (CDO). This paper has shown that the use of structural credit models produces a simple closed-form expression for a basic hybrid derivative. Future research will focus on using structural models to price the more exotic hybrid derivatives that are increasingly becoming available.

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