CoRisk: measuring systemic risk through default probability contagion

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Summary

Systemic Risk

- De Bandt & P. Hartmann (2000):
  A systemic crisis can be defined as a systemic event that affects a considerable number of financial institutions or markets in a strong sense. [...] At the heart of the concept is the notion of **contagion**, *a particularly strong propagation of failures from one institution, market or system to another.*

- Hendricks (2009):
  Systemic risk is the risk of a phase transition from one equilibrium to another, much less optimal equilibrium, characterized by multiple self-reinforcing feedback mechanisms making it **difficult to reverse**.

- Benoit et al. (2015):
  Systemic risk is the risk that many market participants are simultaneously affected by severe losses, which then **spread** *through the system.*

Literature Review

1. Measures of systemic risk for the banking sector:
   - Acharya et al. (2010), Adrian & Brunnermeier (2011), Brownlees & Engle (2012), Acharya et al. (2012), Dumitrescu & Banulescu (2014), Hautsch et al. (2015),
   - Conditional probabilities of default, based on that of the others/system $\longrightarrow$ Bivariate approach, **No contagion effects**;
2. Network models:
   - Billio et al. (2012), Barigozzi & Brownlees (2013), Diebold & Yilmaz (2014), Ahelegbey et al. (2015),
   - Channels of contagion $\longrightarrow$ **No predictions**;
3. Econometric causal methods:
   - Duffie et al. (2000), Lando & Nielsen (2010), Koopman et al. (2012), Betz et al. (2014),
   - Conditional probabilities of default, based on exogenous factors $\longrightarrow$ **No contagion effects**;

Motivation & Objectives (1/3)

- Predictive capability;
- Endogeneity and non-linearity;
- Systematic & Systemic risk;
- Channels of contagion & PD.

CROSS-SECTIONAL DIMENSION + TIME DIMENSION

Multivariate Stochastic Processes

1. Sovereign Risk
2. Corporate Risk
3. Bank Risk

- Linear combination of an idiosyncratic and a systematic factor;
- Spread measure Z_t.

Motivation & Objectives (2/3)

Institution-specific PD

Spread measure $Z_t \longrightarrow$ PD of each economic sector in each country.

+

Partial Correlation Networks

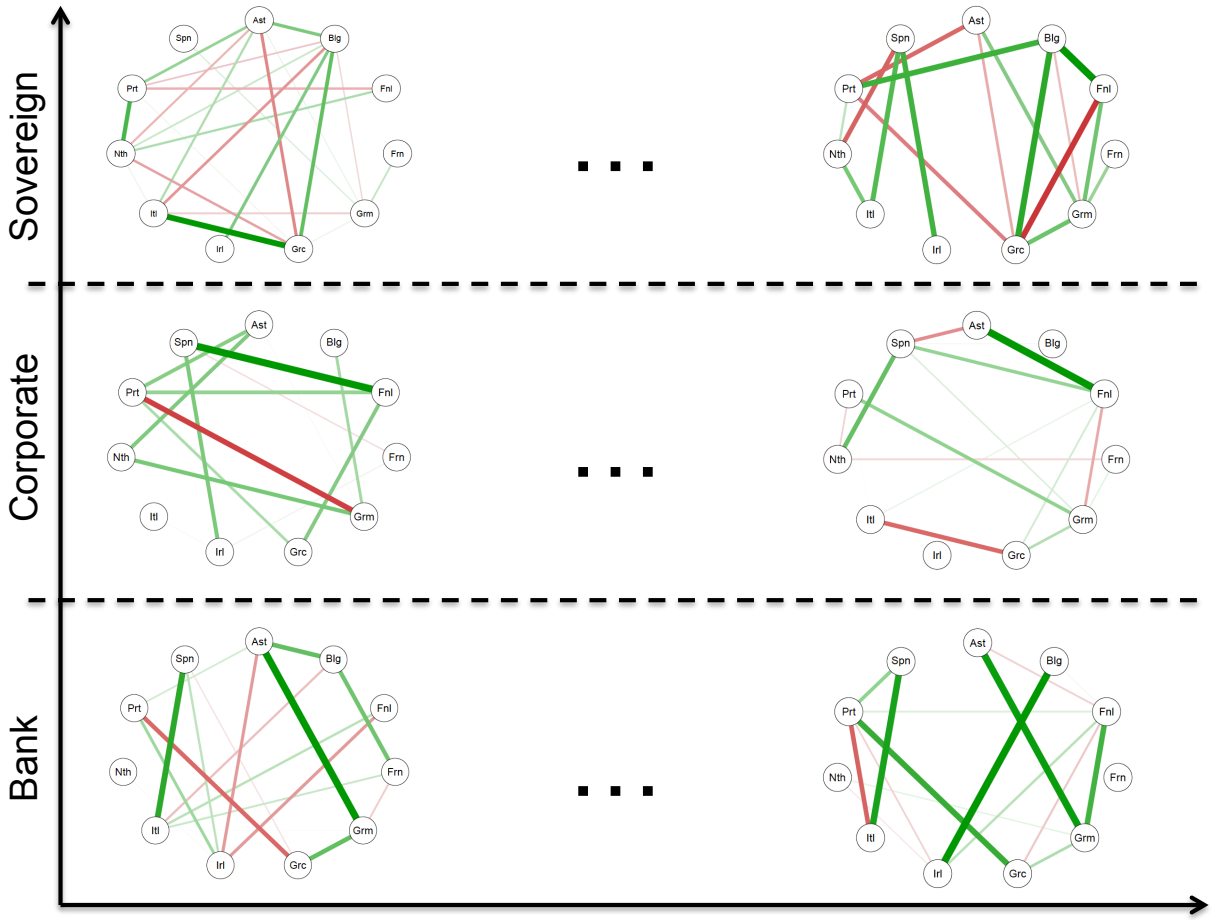
Spread measure $Z_t \longrightarrow$ partial correlation networks for each economic sector.

=

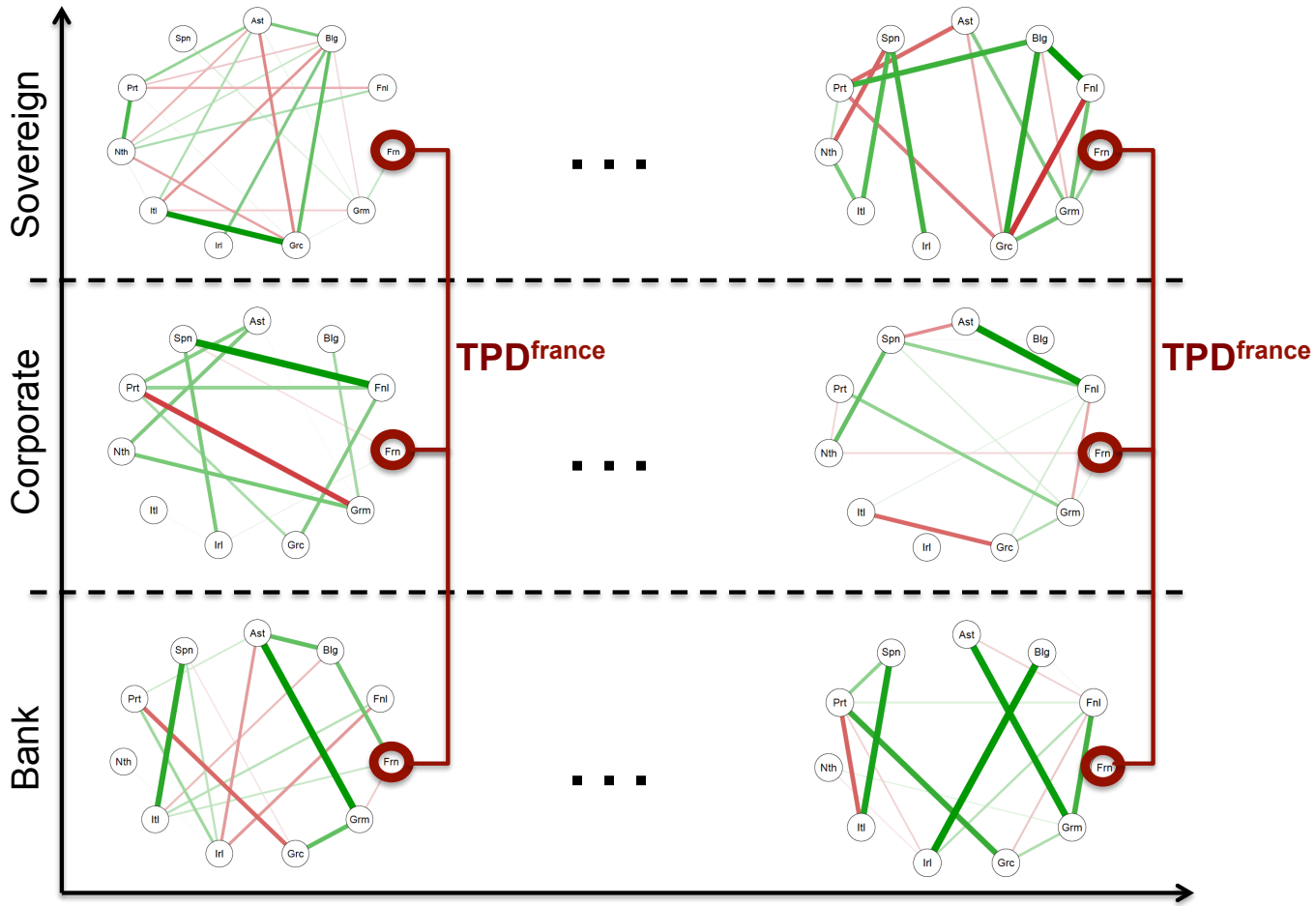
CoRisk

- Change in the PD of each economic sector in each country due to contagion;
- "Total" PD;
- Aggregate PD at the country level.

Motivation & Objectives (3/3)



Motivation & Objectives (3/3)



Sectorial Spreads

- N countries, 1 economic sector;
- For each country $i = 1, ..., N$, we define:

Sectorial Spreads

$$Z_t^i = y_t^i - S_t, \tag{1}$$

$$\begin{cases} \mathrm{d}\, S_t = (a - vS_{t-1})\,\mathrm{d}\, t + b\sqrt{S_{t-1}}\,\mathrm{d}\, B_t, \\ \mathrm{d}\, y_t^i = (\theta_1^i - \theta_2^i y_{t-1}^i)\,\mathrm{d}\, t + \theta_3^i \sqrt{y_{t-1}^i}\,\mathrm{d}\, W_t, \end{cases}$$

Correlation Structure between countries

$$\begin{cases} \mathrm{Corr}[y_t^i, y_t^j] = \rho^{ij}, \\ \mathrm{Corr}[S_t, y_t^j] = \gamma^j. \end{cases} \tag{2}$$

Correlated Sectorial Spreads

- N countries ($V = \{1, ..., N\}$), 3 economic sectors ($W = \{1, 2, 3\}$);
- For the three sectors (1=sovereign, 2=corporate, 3=banking), for $i \in V$ and $\{m, n\} \in V \times W$ we define:

Three Spread Measures

$$\begin{cases} Z^i_{t,1} = y^i_{t,1} - S_t, \\ Z^i_{t,2} = y^i_{t,2} - S_t, \\ Z^i_{t,3} = y^i_{t,3} - S_t, \end{cases} \tag{3}$$

Correlation Structure between Countries and Sectors

$$\begin{cases} \text{Corr}[y^m_t; y^n_t] = \rho^{mn}, \\ \text{Corr}[y^m_t; S_t] = \gamma^m, \end{cases} \tag{4}$$

Covariance Matrix

From the correlation structure between countries and sectors we derive the covariance matrix, which can be decomposed as

$$A = \Phi \cdot \Theta^T; \tag{5}$$

$$[\Phi]^m = \left[\beta^m b\sqrt{S_0}, \quad \beta^m, \quad \alpha^m \sqrt{S_0 y_0^m} b\theta_3^m [\Gamma]^m, \quad \alpha^m \sqrt{y_0^m}\theta_3^m \sqrt{[P]^m}\right],$$

$$[\Theta^T]^n = \begin{bmatrix} \beta^n b\sqrt{S_0} \\ \alpha^n \sqrt{S_0 y_0^n} b\theta_3^n [\Gamma]^n \\ \beta^n \\ \alpha^n \sqrt{y_0^n}\theta_3^n \sqrt{[P]^n} \end{bmatrix}.$$

Partial Correlations & Correlation Network

- From the inverse of the covariance matrix (A^{-1}, elements σ^{mn}), partial correlations can be derived:

$$\rho_{mn|S} = \frac{-\sigma^{mn}}{\sqrt{\sigma^{mm}\sigma^{nn}}}. \tag{6}$$

  Correlations between two elements, conditional on the remaining elements of the system (S).

- Undirected graph $G = (P, E)$ based on partial correlations:
  $P = V \times W = \{1, ..., 3N\}$ vertex set,
  $E = P \times P$ edge set,
  An edge between two nodes m, n is present if and only if the corresponding partial correlation $\rho_{mn|S}$ is significantly different from zero.

Institution-specific PD

- Risk-free context;
- Dynamic of debt = dynamic of risk-free debt;

$$\begin{cases} D^m_{t+1} = (1 - PD^m_t) e^{y^m_t} D^m_t, \\ D^m_{t+1} = e^{S_t} D^m_t. \end{cases} \tag{7}$$

- Default probability of each economic sector in each country, based on the spread measure Z_t:

Institution-specific PD

$$PD^m_t = 1 - e^{-Z^m_t}. \tag{8}$$

From Partial Correlations to TPD

- $\{m, n\} \in (V \times W)$, $S = (V \times W) \setminus \{m, n\}$;
- It can be demonstrated that $|\rho_{mn|S}| = |\rho_{nm|S}| = \sqrt{a_{mn|S} \cdot a_{nm|S}}$, where

$$\begin{cases} Z^m = a_m + \sum_{n \neq m} a_{mn|S} Z^n; \\ Z^n = a_n + \sum_{m \neq n} a_{nm|S} Z^m. \end{cases}$$

Objective

- Is the default probability of node m affected by contagion with neighbours n?
- $PD^m = f(Z^m, t) \longrightarrow TPD^m = f(PD^{n \neq m}, \rho_{mn|S}, t)$

It can be demonstrated that:

TPD

$$TPD_t^m = 1 - (1 - PD_t^m) \cdot \prod_{n \neq m} (1 - PD_t^n)^{\rho_{mn|S}} \tag{9}$$

From TPD to CoRisk

$CoRisk_{in}$

$$CoRisk^m_{in,t} = 1 - \prod_{n \neq m} (1 - PD^n_t)^{\rho_{mn|S}} \tag{10}$$

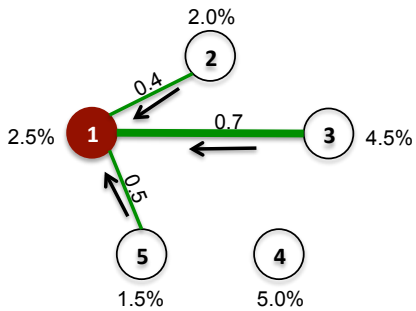
$$CoRisk^m_{in} = \frac{(1 - PD^m) - (1 - TPD^m)}{1 - PD^m}.$$

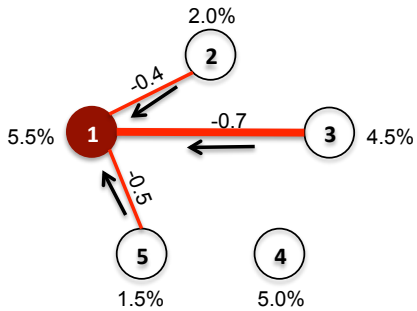
- $CoRisk_{in}$ = change in the survival probability of an agent m when contagion deriving from its first-order neighbours is included.
- To what extent agent m affects its neighbours:

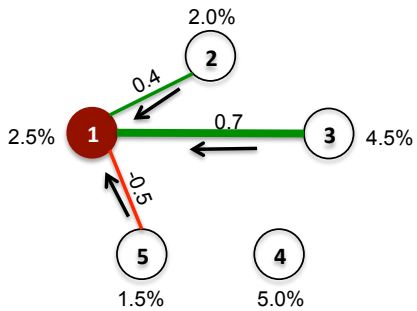
$CoRisk_{out}$

$$CoRisk^m_{out,t} = 1 - \prod_{n \neq m} (1 - PD^m_t)^{\rho_{nm|S}} = 1 - (1 - PD^m_t)^{\sum_{n \neq m} \rho_{nm|S}}. \tag{11}$$

$CoRisk_{in}$ or vulnerability: illustrative example

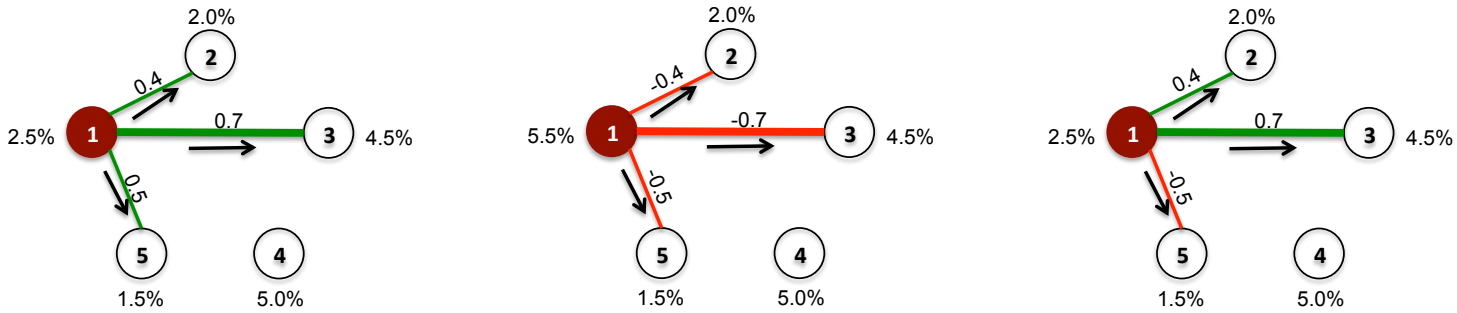






1. $CoRisk_{in} > 0$, $TPD > PD$;
2. $CoRisk_{in} < 0$, $TPD < PD$;
3. $CoRisk_{in} > 0$, $TPD > PD$ but lower than in the first example.

$CoRisk_{out}$ or systemic importance: illustrative example



1. $CoRisk_{out} > 0$;
2. $CoRisk_{out} < 0$;
3. $CoRisk_{out} > 0$, but lower than in the first example.

Aggregated TPD

- TPD of each economic sector $\longrightarrow$ aggregate TPD of each country;

Assumption

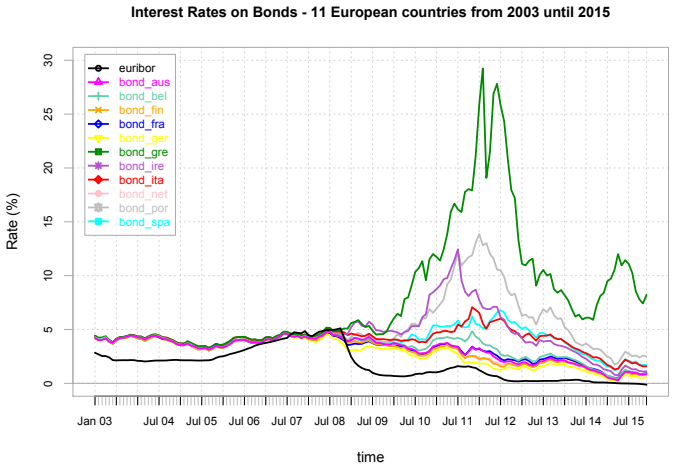
A country defaults if **at least one** of its economic sectors defaults.

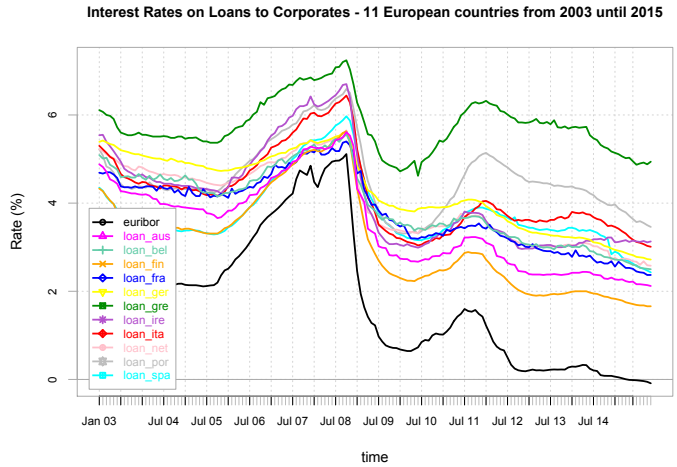
- TPD^m = conditional probability;
- A_1^i, A_2^i, A_3^i = sets of defaults;
- $S^i = \{A^m; \forall m \in V \times W, m \in ne(i,j), m \neq (i,j)\}$
- We are looking for $P(\bigcup_{j \in W} A_j^i | S^i)$

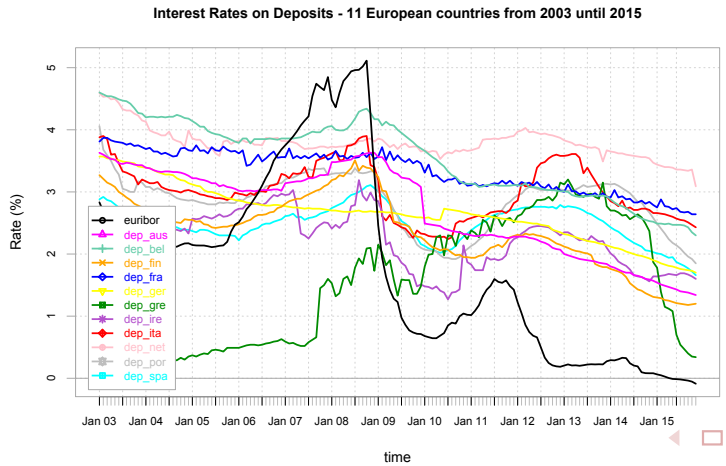
Aggregate TPD

$$TPD^i_{country,t} = 1 - [1 - TPD^i_{1,t}] \cdot [1 - TPD^i_{2,t}] \cdot [1 - TPD^i_{3,t}], \qquad (12)$$

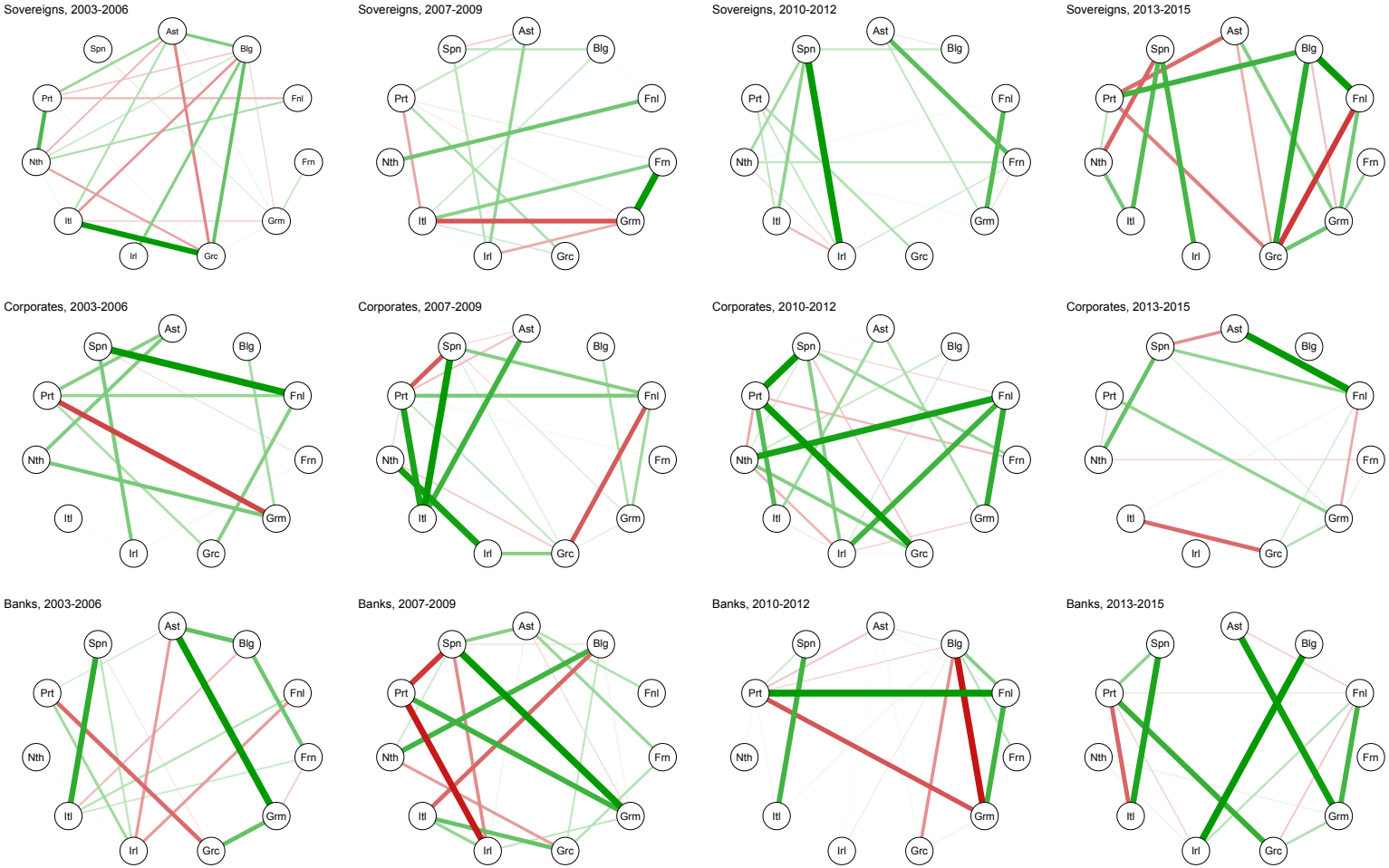
Bonds, Loans & Deposits







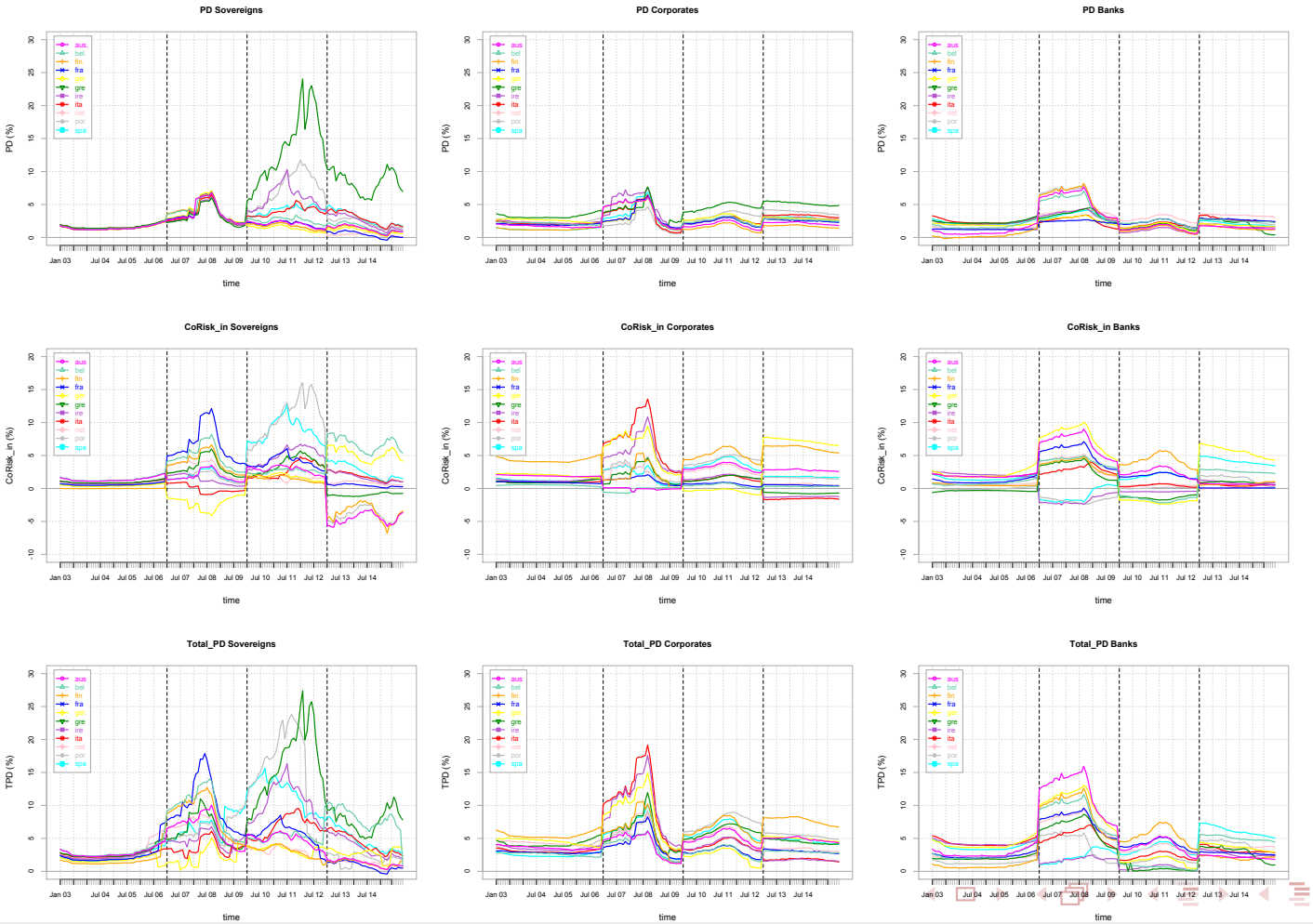
Partial Correlation Networks



Partial Correlation Networks

1. SOVEREIGNS:
   - Financial crisis: number of significant correlations starts decreasing,
   - Sovereign crisis: a clustering effect starts emerging,
   - Post-crisis: two distinct clusters (Core vs Peripheral countries);
2. CORPORATES:
   - Financial crisis: number of significant correlations starts decreasing,
   - Post-crisis: few significant correlations, at the zero-lower-bound the systematic component is not significant;
3. BANKS:
   - Sovereign crisis: sparse network,
   - Post-crisis: a clustering effect starts emerging.

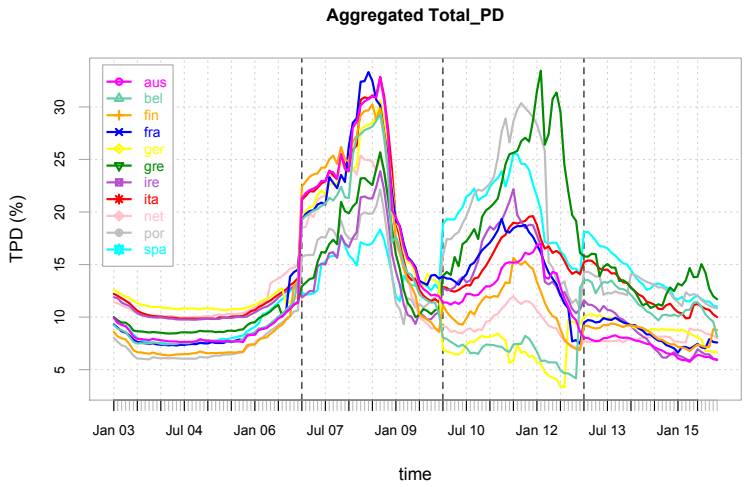
Institution-specific PD, $CoRisk_{in}$ & TPD



Institution-specific PD, $CoRisk_{in}$ & TPD

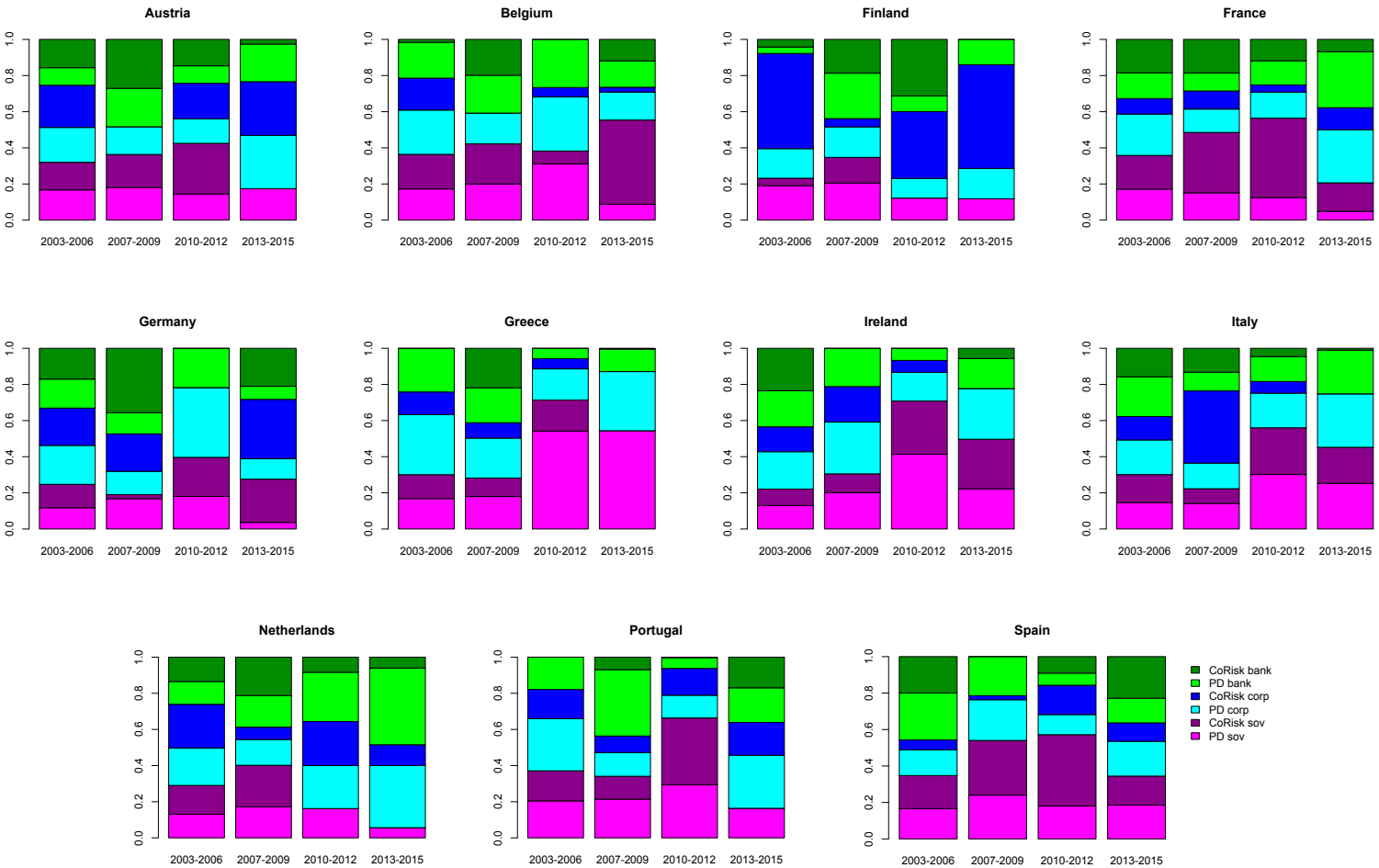
1. SOVEREIGNS:
   - PDs reflect interest rates on bonds,
   - $CoRisk_{in}$: high values for countries positively connected to peripheral ones (France, Belgium),
   - Loop effects: clusters $\longrightarrow$ peripheral countries strongly affected not only by high PD, but also by contagion to each other $\longrightarrow$ TPD becomes higher and higher;
2. CORPORATES:
   - PD less volatile across both countries and time,
   - $CoRisk_{in} > 0$ in all countries during the financial crisis,
   - $CoRisk_{in}$ prevailing effect;
3. BANKS:
   - PDs suffered only the financial crisis,
   - $CoRisk_{in}$ as expected until 2012: positive for core economies (contagion with peripheral countries) and low or negative for peripheral economies,
   - $CoRisk_{in}$ increases during the post-crisis period because of clustering effects.

Aggregated TPD



- Financial crisis: more homogenous impact across countries;
- Peculiarities: France (positive correlations with Italy) and Ireland (deep crisis in 2011 and strong reforms afterwards);
- Pre- vs Post- crisis years: stable and homogenous situation $\longrightarrow$ high volatilities in all countries + two clusters;
- Persisting effect of the sovereign crisis.

Systemic Risk Contributions

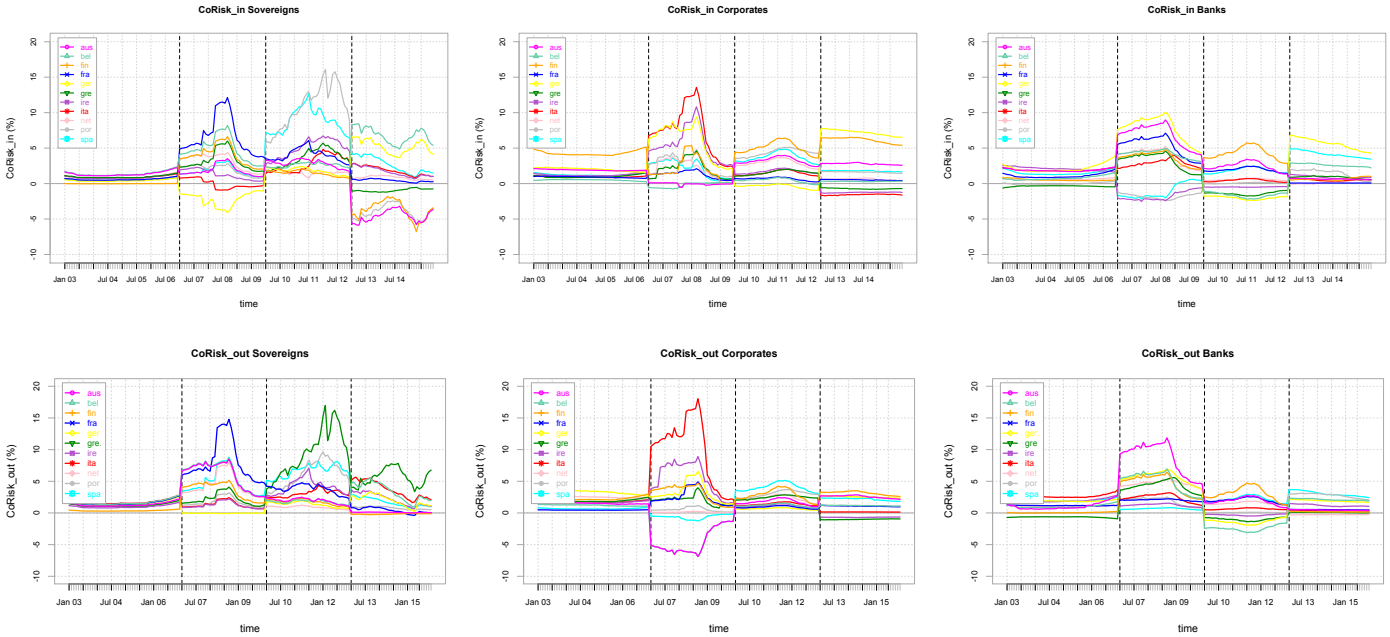


Systemic Risk Contributions

- **Sovereign contribution:** larger in peripheral countries;
- **Corporate contribution:** larger during "normal" times, depending on
  - institution-specific PD (peripheral countries),
  - contagion (core countries);
- **Bank contribution:** high bank CoRisk in core economies during the sovereign crisis, because of exposition to peripheral banks;
- **Sovereign & Bank contributions:** increased during the financial and sovereign crisis;
- **Distribution** of the TPD in its six components: quite homogenous before the financial crisis, but not back to normality in recent years:
  - contagion risk in core economies,
  - high institution-specific PD + clustering effects (loops) in peripheral economies.

$CoRisk_{in}$ vs $CoRisk_{out}$

$CoRisk_{in}$: **vulnerability** vs $CoRisk_{out}$: **systemic importance**



- Pre-crisis and financial crisis: $CoRisk_{in} \sim CoRisk_{out}$, because PDs were homogenous across countries;
- Post-crisis: peripheral countries have highest $CoRisk_{out}$;

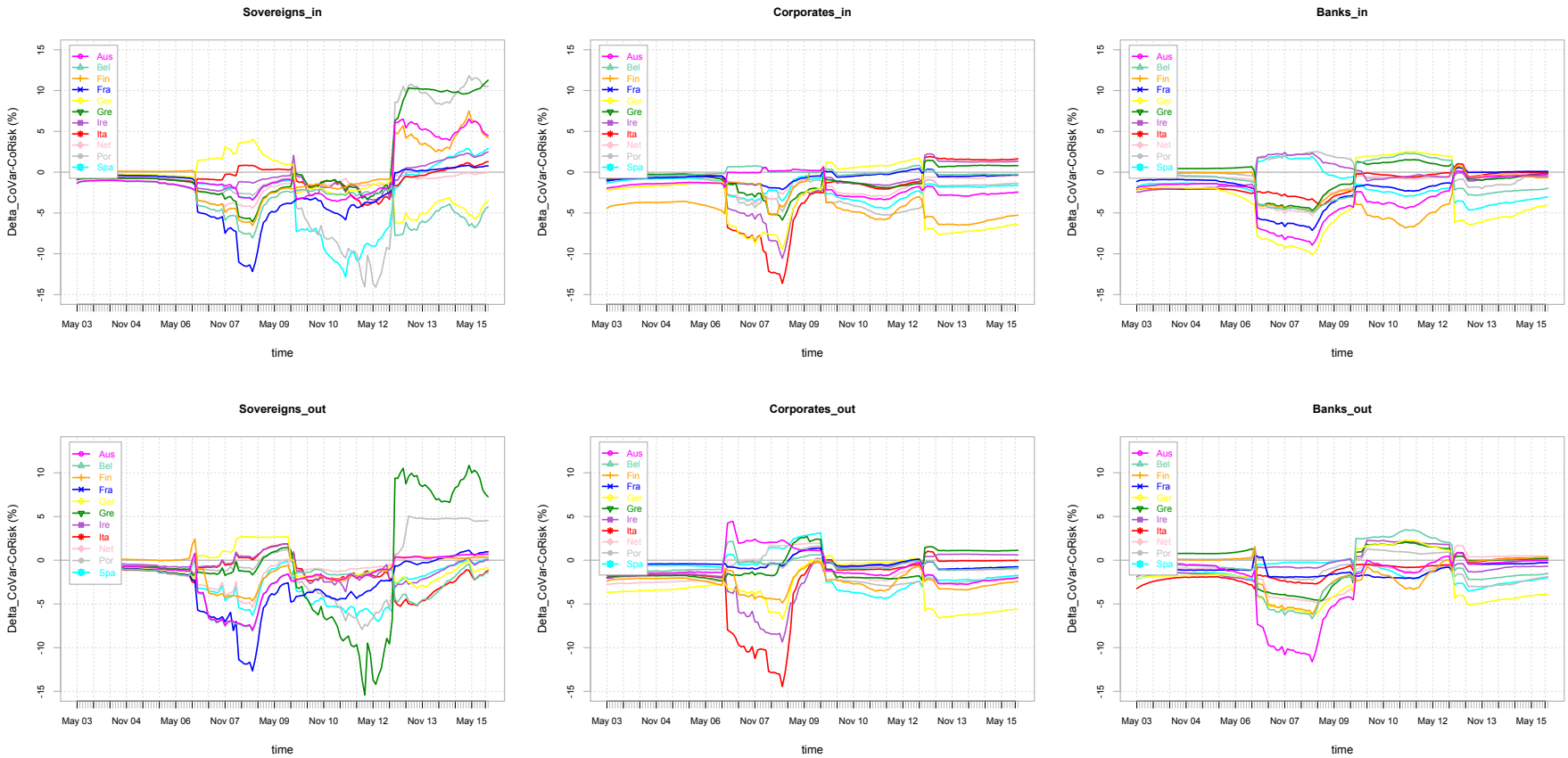
$CoRisk_{in}$ vs Degree of connectivity and Eigenvector centrality

- Comparison of results and rankings;
- Non-parametric Spearman correlation coefficient/test on rankings:

	Sovereign		Corporate		Bank	
Period	*DC*	*Eigen.*	*DC*	*Eigen.*	*DC*	*Eigen.*
2003-2006	0.436	0.136	0.936	0.373	0.764	0.245
2007-2009	0.582	0.064	0.809	0.811	0.936	0.518
2010-2012	0.136	-0.736	0.691	0.655	0.982	0.345
2013-2015	0.736	0.018	0.927	0.245	0.982	0.573

- **CoRisk vs DC**: CoRisk weights each link considering partial correlation + **PD of neighbours**;
- **CoRisk vs Eigen.**: Eigenvector centrality considers the importance of each node according to its links to important nodes, without considering PD of neighbours $\longrightarrow$ amplification of the distance between the two measures (especially during crisis years).

$CoRisk_{in}$ and $CoRisk_{out}$ vs $\Delta CoVar$



Conclusions

1. SOVEREIGN RISK:
   - Larger in peripheral than in core countries,
   - Core countries: mainly due to contagion (CoRisk),
   - Peripheral countries: due both to contagion (loop effects) and institution-specific PD;

2. CORPORATE RISK:
   - Most important risk during "normal" times,
   - Core countries: mainly due to contagion (CoRisk),
   - Peripheral countries: mainly due to institution-specific PD;

3. BANK RISK:
   - High in all countries during the financial crisis,
   - Core countries: mainly due to contagion (CoRisk) during the financial crisis (exposition to peripheral banks),
   - Peripheral countries: due both to contagion (loop effects) and institution-specific PD.

Conclusions

Conclusion 1

Sovereign crisis (clustering $\rightarrow$ loop effects)
greater than
Financial crisis (homogenous across countries)
$\downarrow$
Post-crisis period: distribution of risk not homogenous in its six components

WHY?
Different reactions to the financial crisis: peripheral countries (high public debt), no fiscal space, imbalances emerged.

Conclusion 2

$CoRisk_{in}$ **vs** $CoRisk_{out}$

- Peripheral countries: systemically importance prevails
- Core countries: vulnerability prevails