Appendix: Oil prices and carbon taxes

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Abstract

A global carbon tax might be expected to raise the price of fossil fuel and depress demand and hence GHG emissions. This note shows that the incidence of a carbon tax will be partly, and in extreme cases, wholly, shifted on to the rent of fuel producers, reducing the rise in post-tax price and in extreme cases exacerbating climate damage (Green Paradox). The practical question is to determine the magnitude of this shifting and consequential emissions rebound for an efficiently set carbon tax, which should initially be rising at about the rate of interest.

1 Introduction

This note provides a simple Hotelling (1931) model of competitive exhaustible resource pricing to illustrate the interaction of carbon taxes and oil prices. It can be elaborated to provide more realistic results, and Mejean and Hope (2010) illustrate the results of more careful modelling exercises for the oil market that also allows for uncertainty. A full study of climate change policy would also examine other fossil fuels, particularly coal, whose resource base appears substantially larger than the absorptive capacity of the atmosphere (Allen et al, 2009; Meinshausen et al, 2009) but whose exploitation can be managed by a universal carbon tax or price.[1] Oil is nevertheless important as it is hard to replace as a transport fuel, given the limited resource base for biofuels. As oil is depleted, so its price will rise until eventually it will reach a level at which some substitute or backstop alternative becomes competitive and can replace the oil as the last barrel is extracted.

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[1] More coal could be burned if emissions were captured and sequestered.

The questions that can be asked of this simple model are how the carbon tax affects the pre and post-tax price of oil, how that depends on the carbon intensity of the backstop alternative, and what impact the carbon tax on cumulative emissions.

2 Competitive oil pricing theory

The simplest theory of oil pricing assumes perfect competition between oil producers, a known total stock of oil, S_0, at date 0, a common and constant unit extraction cost, m, with demand growing at rate g, but with a constant price elasticity of demand of ε (taken as a positive number). There is assumed to be a perfect substitute from a perfectly elastic backstop technology (oil sands, hydrogen from nuclear fusion, solar PV of CSP) at a price $\overline{p}$, perfect certainty, no risk, no technical change, and a common discount rate r. Demand at date t is $q_t(p_t) = Ae^{gt}(p_t)^{-\varepsilon}$ and if $m < \overline{p}$, then total oil must be exhausted at date T when the price reaches the backstop level and thereafter oil is replaced by its perfect backstop substitute:

$$\int_0^T q_t(p_t)dt = S_0. \tag{1}$$

If there is no constraint on the instantaneous rate of oil depletion, arbitrage between keeping oil in the ground and selling it now means that at date t the rent $p_t - m$ must be growing at rate r, so that

$$p_t = m + (\overline{p} - m)e^{r(t-T)}. \tag{2}$$

In this simplest of all models, extraction costs are assumed to be zero, so that

$$p_t = p_0 e^{rt} = \overline{p}e^{r(t-T)}, \tag{3}$$

and the date of exhaustion will satisfy:

$$\int_0^T q_t(p_t)dt = S_0 = A\int_0^T \left(\overline{p}e^{r(t-T)}\right)^{-\varepsilon} e^{gt}dt, \tag{4}$$

$$S_0 = Ae^{\varepsilon rT}\overline{p}^{-\varepsilon}\psi(T), \quad \psi(T) = \int_0^T e^{-(\varepsilon r - g)t}dt = \left(\frac{1 - e^{-(\varepsilon r-g)T}}{\varepsilon r - g}\right). \tag{5}$$

This allows the time to depletion, T, and hence the initial price p_0 to be determined. It is then relatively simple to estimate the response of the present price to changes in total stocks, the discount rate, r, the growth rate of demand, g, the demand elasticity, ε, and the backstop price, $\overline{p}$. For example, from (3),

$$\frac{\partial p_0}{p_0 \partial r} = -(T + r\frac{\partial T}{\partial r}),$$

where $\partial T/\partial r$ can be found by differentiating (5) with respect to r.

2.1 Competitive pricing with a sequence of fields

In a competitive world with no supply constraints cheap oil would be exhausted before more expensive oil, and the economic reserves would be that amount of oil that can be extracted at a cost less than the backstop cost. Suppose that the stocks of oil with unit extraction costs m_i at date 0 are S_i, $i = 1, ..n$, and that each reservoir will be exhausted at date T_i. Then the values of T_i will satisfy a sequence of conditions, working back from the final price:

$$\int_{T_{n-1}}^{T_n} q_t(p_t)dt = S_n = A\int_{T_{n-1}}^{T_n} \left(m_n + (\overline{p} - m_n)e^{r(t-T)}\right)^{-\varepsilon} e^{gt}dt. \tag{6}$$

Given T_n this equation will determine T_{n-1} and hence $p_{T_{n-1}} \equiv \overline{p}_{n-1}$, which in turn defines the previous price trajectory, so that in general

$$p_t = m_j + (\overline{p}_j - m_j)e^{r(t-T_j)},\ j = 1, ...n,\ \overline{p}_n = \overline{p}, \tag{7}$$

where the transition prices $\overline{p}_j$ are determined as the initial price at which the next most expensive field is first tapped. The first stock condition will give another equation for T_1 that will allow T_n to be determined.

3 Carbon pricing

Greenhouse gases are global persistent stock public bads - that is, they are non-excludable (inividuals cannot choose their own level of global GHG concentrations) and emissions by any affect all. In an ideal world the damage done by each extra tonne emitted would be taxed or charged to encourage efficient emissions abatement. Suppose that emissions at date t are e_t in tonnes carbon equivalent (tCe) and the stock of GHG at that date is C_t tCe, which evolves as

$$\frac{dC_t}{dt} \equiv \dot{C} = \gamma e_t - \mu C, \tag{8}$$

where γ is the fraction of CO_2 not absorbed relatively quickly by the biosphere, and μ is the net rate of absorption by other carbon sinks. At present ecosystems are estimated to absorb about half of anthropogenic CO_2 emissions quite rapidly (oceans about 24% and land about 30%, Munang et al, 2009), but this absorptive capacity is declining at about 1% per year, partly because the absorptive capacity of the ocean depends on the concentration difference between the atmosphere and the ocean, and will decrease as ocean concentrations rise relative to the atmosphere. Earlier estimates[2] of the global carbon cycle between 1992-97 suggest that fossil fuel and cement emissions (but excluding the considerable contribution of farming and land

[2] at http://cdiac.ornl.gov/pns/graphics/c_cycle.htm

use change) accounted for 6.2GtC/yr,[3] but the atmospheric stock of 775 GtC was only then increasing by 3.8 GtC/yr, consistent with about half of emissions being rapidly absorbed. The proper accounting for CO_2 emissions is thus more complicated than suggested by equation (8) in that half would be removed within a year or so, about one-quarter is very persistent and can be treated as permanent, while the remaining quarter is absorbed over a period of 50-100 years,[4] requiring a distinction between permanent and decaying CO_2 as follows:

$$\begin{aligned}\dot{C}^p &= 0.25e_t,\\ \dot{C}^d &= 0.25e_t - \mu C^d,\\ C_t &= C_t^p + C_t^d = Ae^{-\mu t} + \frac{2+\mu}{4(1+\mu)}E_t,\\ \dot{C} &= \frac{2+\mu}{4}e_t - \mu C.\end{aligned}$$

In these equations, E_t is cumulative emissions to date, $\dot{E} = e_t$, so as a rough approximation valid over time scales of decades, equation (8) can still be used with $\mu \simeq 1\%$, and $\gamma \simeq \frac{1}{2}$.

If global climate change damage at date t, D_t, depends on the prevailing stock of atmospheric carbon, C_t, $D_t = D(C_t)$, and if the instantaneous net benefit (gross benefit less the cost of abatement) of emissions is $B(e_t)$, then the optimal rate of emissions would maximize the present discounted value of these benefits less the climate change damage:[5]

$$J = \int_0^T (B(e_t) - D(C_t))\, e^{-rt}dt,$$

subject to (8) and the initial stock of carbon, C_0. This is a standard control problem to be maximized by choosing the time path of e_t. The Hamiltonian is

$$\begin{aligned}\overline{H} &= (B(e_t) - D(C_t))\, e^{-rt} + \lambda\dot{C},\\ \dot{\lambda} &= -\frac{\partial\overline{H}}{\partial C} = \sum_i \frac{dD_i}{dC}e^{-rt} + \mu\lambda.\end{aligned}$$

It is convenient to work in present value terms, and to work with a tax (negative price) rather than the shadow price, λ, so let $H = \overline{H}e^{rt}$ and $\tau = -\lambda e^{rt}$. Hence $\dot{\tau} = r\tau - \dot{\lambda}e^{rt}$ and

$$\begin{aligned}H &= (B(e_t) - D(C_t)) - \tau\left(\gamma e_t - \mu C\right),\\ \dot{\tau} &= \frac{\partial H}{\partial C} + r\tau = -\frac{dD}{dC} + (\mu + r)\tau, &\quad (9)\\ \frac{\partial H}{\partial e_t} &= 0 = B'(e_t) - \gamma\tau, &\quad (10)\end{aligned}$$

[3] compared to presumably total anthropogenic emissions of around an average of 11 GtC between 2000 and 2006 (Meinshausen et al, 2009)

[4] That is the way atmospheric CO_2 is modelled in the PAGE model of Hope (2006) used in Stern (2006).

[5] Ulph and Ulph (1994) adopt a similar approach to finding the time path of a carbon tax.

where the last equation comes from the maximizing choice of emissions. It shows that the optimal emissions path can be decentralized by charging a carbon tax at rate $\gamma\tau_t$, so that emitters will maximize net benefit, $B(e_t) - \gamma\tau_t e_t$, providing the tax grows at rate

$$\frac{d\tau}{\tau dt} = r + \mu - \frac{D'}{\tau}. \tag{11}$$

In early years the instantaneous marginal damage D' of another tonne of carbon might be modest and might roughly counterbalance μ, so that carbon tax rate should grow at roughly the rate of interest - which would be the natural outcome of a cap-and-trade system with banking, such as the second phase of the European Emissions Trading Scheme.

4 The effect of a carbon tax on the oil price

Now suppose that a global carbon tax is imposed on the carbon content of fuels, set initially at the level of τ per tonne CO_2, but growing at rate δ, where δ is given by (11) and will be assumed moderately constant, so that at date t the tax is $\tau e^{\delta t}$. The tax is imposed on carbon emissions and hence paid by the oil or substitute consumers, and the oil producers receive the pre-tax price, p_t. Suppose the carbon content of oil is α tonnes CO_2 per unit of oil, and that of the backstop is β, where β might be considerably larger than α in the case of unconventional oil, but zero (zero-C) in the case of hydrogen from fusion, PV or photosynthesis. If all extraction costs are zero, the tax-inclusive price of oil at date t will be $P_t = p_0 e^{rt} + \alpha\tau e^{\delta t}$, but the tax-inclusive backstop price at the date of exhaustion, T_c, (which we can expect to differ from the zero tax exhaustion date, T) will be $\overline{P} = \overline{p} + \beta\tau e^{\delta T_c}$. The pre-tax price received by oil producers will be p_t and this will be determined by the stock exhaustion condition that the tax-inclusive oil price reaches the tax-inclusive backstop price at date T_c with $P_{T_c} = \overline{P}$.

Given a global carbon tax, the tax-inclusive price of oil will again be determined by the exhaustion condition and the tax-inclusive backstop price $\overline{P} = \overline{p} + \beta\tau e^{\delta T_c} = p_0 e^{rT_c} + \alpha\tau e^{\delta T_c}$. Then

$$\begin{aligned} p_0 e^{rT_c} &= \overline{p} + (\beta - \alpha)\tau e^{\delta T_c}, \\ p_t &= (\overline{p} + (\beta - \alpha)\tau e^{\delta T_c})e^{r(t-T_c)}, \qquad (12) \\ P_t &= p_t + \alpha\tau e^{\delta t}, \\ P_t &= (\overline{p}e^{-rT_c} + (\beta - \alpha)\tau e^{(\delta - r)T_c})e^{rt} + \alpha\tau e^{\delta t}. \qquad (13) \end{aligned}$$

Note that if the carbon tax rises at the rate of interest, so that $\delta = r$, then

$$P_t = (\overline{p}e^{-rT_c} + \beta\tau)e^{rt}.$$

If the backstop is zero-carbon, so that $\beta = 0$, then the tax-inclusive price of oil follows the same trajectory as the no-tax oil price case in (3), and hence the date of exhaustion is unchanged, $T_c = T$, and so is the initial tax-inclusive price. We can summarize this in

Proposition 1 *If oil extraction costs are zero and the backstop technology is carbon neutral, then a global carbon tax that rises at the rate of interest will be entirely borne by competitive oil suppliers, with no impact on the date of exhaustion and the consumption price of oil.*

An implication of this is that the pre-tax price will be lowered by the full amount of the carbon tax, so while taxing countries will consume the same amounts of oil as before, countries evading the tax will enjoy cheaper oil and will increase consumption, leading to higher emissions and more rapid depletion - the Green Paradox (Sinn, 2008; van der Ploeg and Withagen, 2010). This is readily established. Suppose that a fraction of total demand, θ, comes from countries that fail to impose the carbon tax, so that total demand is given by

$$q_t = Ae^{(g-r\varepsilon)t}\left((1-\theta)(p_0+\tau)^{-\varepsilon} + \theta p_0^{-\varepsilon}\right).$$

The initial price $p_0 = \overline{p}e^{-rT} - \tau$ is determined by the exhaustion condition (1) so that

$$\int_0^T q_t(p_t)dt = S_0 = A\psi(T)\left((1-\theta)(\overline{p}e^{-rT})^{-\varepsilon} + \theta(\overline{p}e^{-rT} - \tau)^{-\varepsilon}\right). \tag{14}$$

Totally differentiate the log of (14) w.r.t. T holding θ constant:

$$\begin{aligned} 0 &= \frac{e^{-(r\varepsilon-g)T}}{\psi(T)} + \frac{\varepsilon r(1-\theta)(\overline{p}e^{-rT})^{-\varepsilon} + \varepsilon\theta(\overline{p}e^{-rT}-\tau)^{-\varepsilon-1}(r\overline{p}e^{-rT} + d\tau/dT)}{(1-\theta)(\overline{p}e^{-rT})^{-\varepsilon} + \theta(\overline{p}e^{-rT}-\tau)^{-\varepsilon}}, \\ &= r(1-\theta)(\overline{p}e^{-rT})^{-\varepsilon} + \theta(\overline{p}e^{-rT}-\tau)^{-\varepsilon-1}(r\overline{p}e^{-rT} + d\tau/dT) \\ &\quad + \frac{e^{-(r\varepsilon-g)T}((1-\theta)(\overline{p}e^{-rT})^{-\varepsilon} + \theta(\overline{p}e^{-rT}-\tau)^{-\varepsilon})}{\varepsilon\psi(T)}. \end{aligned}$$

All the terms except $d\tau/dT$ are positive, and so $dT/d\tau < 0$. Thus

Proposition 2 *(The Green Paradox) Given the assumptions of Proposition 1, any evasion of the carbon tax will result in higher emissions and more rapid depletion of oil and hence release of CO_2 than in the case of no carbon tax.*

More generally, the exhaustion condition pins down the price trajectory by determining the exhaustion date, T_c. Again, if extraction costs are zero,

$$\int_0^{T_c} q_t(P_t)dt = S_0 = A\int_0^{T_c}\left((\overline{p}e^{-rT_c} + (\beta-\alpha)\tau e^{(\delta-r)T_c})e^{rt} + \alpha\tau e^{\delta t}\right)^{-\varepsilon} e^{gt}dt. \tag{15}$$

If the backstop technology has the same carbon intensity as oil, so that $\alpha = \beta$, then regardless of the rate of increase of the carbon tax, the pre-tax price of oil will be given from (12) by

$$p_t = \overline{p}e^{r(t-T_c)},$$

which has the same form as (3). However, it is the post-tax price P_t that determines the exhaustion date, which, from (15) will be the solution to

$$S_0 = A\int_0^{T_c} \left((\overline{p}e^{-rT_c}) + \alpha\tau e^{(\delta-r)t} \right)^{-\varepsilon} e^{-(\varepsilon r-g)t}dt. \tag{16}$$

It is then easy to argue that the date of exhaustion will be delayed as a result of the carbon tax, as follows. If we assume $T_c = T$, then the integral of the first bracket in (16) is less than the first term in (5) so $\psi(T_c) > \psi(T)$ or $T_c > T$. The same argument applies if $\beta \geq \alpha$, and indeed provided $\beta > 0$. Summarizing

Proposition 3 *If oil extraction costs are zero and the backstop technology is carbon intensive, then a fully effective global carbon tax will delay the date of exhaustion, and hence lower the post-tax price of oil at each date (compared to the case in which the exhaustion date is not altered), with oil consumers and producers sharing the burden of the carbon tax.*

Intuitively, the higher is the carbon intensity of the backstop, the higher must the post-tax oil price be at the date of exhaustion, which reduces demand at each moment before the exhaustion date, delaying the exhaustion date, and thereby tending to lower the initial post-tax price compared to the case in which the exhaustion date is not altered but the entire tax is borne by consumers. In terms of incentives for countries to avoid the carbon tax (the revenue from which we can assume they keep), pre-tax price is lowered by the carbon tax as before, giving the same incentive to avoid imposing the tax, but the reduction in price compared to no carbon tax is smaller.

4.1 Extensions to many fields

If there are a sequence of increasingly costly fields, then the earlier approach can be deployed to solve for the price trajectory. Consider the benchmark case in which there is a common discount rate, so $\delta = r$, so the prices and demand trajectory on the final oil field are

$$\begin{aligned} p_t &= m_n + (\overline{p} + (\beta - \alpha)\tau e^{rT_c} - m_n)e^{r(t-T_c)}, \\ P_t &= m_n + (\overline{p} - m_n)e^{r(t-T_c)} + \beta\tau e^{rt}, \\ S_n &= \int_{T_{n-1}}^{T_n} q_t(P_t)dt. \end{aligned}$$

Again, given the reservoir amounts S_i and the other parameters, the various transition dates T_j, the transition prices $\overline{p}_j$ as in (7), and ultimately the initial oil price can be found as a function of the initial carbon tax, τ. One can then ask how the carbon tax affects the pre- and post-tax price of oil now, and hence what effect if has on the rate of use. Algebraically, this would require solving the sequence of equations

$$\frac{\partial p_0}{\partial \tau} = e^{-rT_1}(\frac{\partial \overline{p}_1}{\partial \tau} - r(\overline{p}_1 - m_1)\frac{\partial T_1}{\partial \tau}),$$

where the $\partial T_j/\partial \tau$ are found from the reservoir stock equations. In practice numerical methods are likely to be simpler to implement.

Mejean and Hope (2010), in a more fully articulated model that allows for varying but uncertain extraction costs, learning by doing, as well as uncertainty about the various parameter values ($g,\ r,\ \varepsilon,\ \delta$), estimates that between 81% and 99% of the carbon tax will be added to the oil price, which will therefore fall by 1% to 19% compared to the no-tax case. Thus a carbon tax with a reasonably global coverage should reduce oil emissions at each date appreciably.

5 The effect of a carbon tax on total oil depletion

If the backstop technology is carbon neutral, then a carbon tax may make the marginal field uneconomic, as its tax inclusive cost may be above that of the backstop. That would decrease the cumulative stock of CO_2 released from oil depletion, but not if the marginal field were still sufficiently cheaper than the backstop cost. If the backstop were at least as carbon-intensive as oil ($\beta \geq \alpha$) then all previously economic oil reserves would be extracted (over a longer time period), and if we ignored the absorption of atmospheric CO_2, the carbon tax would have no impact on oil's contribution to ultimate global warming (although the delay would still have value in deferring disaster). If the initial backstop is carbon-intensive, and if we are serious about limiting cumulative emissions, then at some stage the tax-inclusive price will have to rise to the level at which demand for carbon-intensive fuels falls to zero (with a transition to some zero carbon substitute). In that case we are back in the world of a zero-C backstop, and the main reason for a carbon tax is to ensure that the reserves that are still economic with a carbon tax do not exceed the absorptive capacity of the atmosphere, say C tonnes of CO_2. If the total stock of oil is $S = \sum S_i$, and $\alpha S < C$, then at most $(C - \alpha S)/\beta$ tonnes of oil equivalent of the backstop can be depleted before moving to the zero-C backstop. The cost of the backstop will determine the required final oil price, the date of switching to the zero-C backstop, the final carbon tax, and hence the initial carbon tax. If the cost of this zero-C substitute falls over time the calculation will be slightly more complicated but determined by the same conditions.

5.1 Complications - the carbon cycle

As noted above, atmospheric CO_2 is not currently in equilibrium and is being absorbed at a (rather slow) rate, perhaps 0.25 of 1% per year or about 2-3 GtC/yr. Thus there are obvious benefits to delaying fossil fuel use, as more CO_2 might thereby be absorbed naturally, increasing the total amount that can be exploited before the atmospheric carbon budget is exhausted. However, the effect is likely to be small - spreading fuel use over an extra 20 years might allow an extra 40-60 GtC, only 10% of Allen's estimate of the absorptive capacity of another 500 GtC to limit the chance of a temperature rise of more than 2^0 C to less than 50%.

6 Conclusions

Carbon taxes are intended to discourage greenhouse gas emissions, most of which come from burning fossil fuel. Two factors may tend to counter this desirable reduction. First, the pre-tax price of the fossil fuel may be reduced, leading to a lower post-tax price than intended, and hence a higher rate of GHG emissions, and second, the difference between the pre- and post-tax price provides an incentive for countries to resist imposing the carbon tax within their own jurisdictions. Both of these encourage a rebound effect, in which emissions rebound from the intended reduced level.

These effects are most likely for the case of oil, whose reserves are unlikely to sustain projected business-as-usual demand levels for more than a decade or so, and in the extreme case in which carbon taxes rise at the same rate as oil prices to the cost of a carbon-neutral backstop technology, both effects are extreme - the post-tax oil price will not increase compared to no carbon taxes, but the pre-tax price will fall by the amount of the carbon tax, leading to the strongest rebound effect. Fortunately, in more plausible cases, the rebound appears to be considerably smaller, although the incentive to free-ride by not imposing carbon taxes remains as high as before (and almost equal to the carbon tax, given the modest contribution any deviant would have on its own climate damage).

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