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Supply Function Equilibria: Step functions and continuous representations

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Electricity liberalisation creates electricity markets, whose prices provide the incentive to offer capacity and to invest in new plant. The aim of market design (and market monitoring) is to deliver an efficient, competitive and sustainably reliable supply of electricity, with adequate opportunities to hedge risk. Regulators monitor the markets to detect and prevent abuse, examine proposed mergers, adjudicate rule changes, and assess the adequacy of the reserve margin. Most of these oversight or regulatory functions require an ability to predict market behaviour and wholesale price formation under various counterfactuals – what might happen if a merger is waived through, the rules are changed, capacity payments introduced or abolished, or an interconnector constructed.

This paper reconciles two apparently very different approaches to finding the equilibrium prices in electricity markets, and thus fills an embarrassing gap between theory and reality. The issue is important – large sums of money are devoted to constructing and running market equilibrium models. Some are quite rudimentary in their conception, such as Cournot oligopoly models in which each producer chooses a single level of output at each moment and the market operator finds the market clearing price. More sophisticated models take account of the non-convexities associated with start-up costs, ramp rates, etc (e.g. Powersym developed by the Tennessee Valley Authority) but at the cost of assuming perfect competition. Yet others try and develop intellectually coherent imperfectly competitive models that attempt to find equilibrium prices, but make assumptions that appear at variance with reality, and it is for such models that our paper reconciles two apparently incompatible approaches to finding equilibria.

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The leading theory assumes that generating companies offer a smooth and continuous supply function, specifying the amounts they are willing to supply at each price. The market operator aggregates these supplies and sets the Market Clearing Price at the lowest price at which supply is equal to demand. Generators choose their offers by optimising against the resulting smooth residual demand, which gives a well-defined unique profit maximising solution. In reality, most wholesale markets require offers to take the form of a step function, and the resulting residual demand facing any generator is also a step function, no longer smooth and presenting delicate problems for finding the optimal response. Some economists have therefore argued that the market should be modelled as a discrete unit auction, in which suppliers offer each unit at a constant price. In many of the examples analysed, if one supplier could predict the price at which other suppliers would offer their units, they could slightly undercut them and hence displace them from the market. If demand varies and there is limited capacity, then the only solution may be a mixed strategy in which suppliers randomly choose the prices at which they will offer their units, leading to unstable prices. Such multi-unit auctions are very hard to solve, and only simple (and possibly misleading) special cases have been presented in the literature.

Other economists have argued that as there are many generating units in most markets (more than 200 in Britain), the steps can be made small enough for the residual demand to be treated as smooth (and hence differentiable) to find stable profit maximising solutions. These different approaches can lead to dramatically different solutions for the equilibrium prices. It is the purpose of this paper to demonstrate that in a well-defined sense it can be legitimate to approximate step-functions by smooth differentiable functions, and hence to draw on the well-developed theory associated with continuous supply functions leading to stable solutions. To prove this result, we develop a new discrete model that has a pure-strategy equilibrium, which can also be useful for solving other problems. For example, it has the potential to enhance the accuracy in econometric studies of bidding in auctions, as our discrete model sidesteps the problem of how to smooth residual demand curves, which has been a somewhat arbitrary and therefore disputed process in previous empirical studies of electricity auctions.

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