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EPRG Working Paper 0917

Cambridge Working Paper in Economics 0932

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JEL Classification H23, Q28, Q54, Q55, Q58

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Publication June 2009

Financial Support TSEC 2

www.electricitypolicy.org.uk

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December 2008
Revised: June 2009

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This paper examines the effects of firm-level innovation in carbon-abatement technologies on optimal cap-and-trade schemes with and without price controls. We characterize optimal cap-and-trade regulation with price cap and price floor, and compare it to the special cases of pure taxation and simple emissions cap. Innovation shifts the tradeoff between price- and quantity-based instruments towards quantity-based emissions trading schemes. More specifically, an increase in innovation effectiveness lowers the optimal emissions cap, and leads to relaxed price controls unless the slope of the marginal environmental damage cost curve is small. Because of the decrease in the emissions cap, innovation in abatement technologies can lead to a higher expected carbon price, so as to provide sufficient incentives for private R&D investments. The expected carbon price decreases once innovative technologies are widely used.

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1 Introduction

In order to bound global warming, the Intergovernmental Panel on Climate Change (IPCC) noted that worldwide annual carbon emissions need to be cut approximately in half by 2050 (IPCC 2008). A mix of different policy instruments is likely to be required to deliver the necessary emissions reductions, including a price for carbon, incentives for technological innovation, and suitable administrative procedures (Stern 2007). We focus on the role of a carbon price, and explore the role of price- and quantity-control instruments on mitigation efforts and investment in improvement of emissions-abatement technologies. We first characterize optimal cap-and-trade schemes with price controls (of which pure taxation and standard cap-and-trade are special cases) and then examine how a change in innovation effectiveness influences the design of carbon markets. We find that an increase in innovation effectiveness can counter-intuitively lead to higher carbon prices, which stems from the fact that in order to encourage technological innovation, the welfare-maximizing regulator may opt to aggressively decrease the emissions cap, leading to a higher expected carbon price despite the anticipated decrease in abatement cost. Because of the increased importance of the emissions cap as a policy instrument, more innovation tends to favor quantity-based instruments over price-based instruments such as taxes: unless the slope of the marginal environmental damage cost curve is small (and an inverse result obtains), an increase in innovation effectiveness will lead to looser price controls (i.e., lower price floor and higher price cap), tipping the scales more towards a quantity-control scheme.

There are three main reasons for a simultaneous consideration of several regulatory instruments.[1] First, a joint optimization of several instruments (prices and quantities) cannot do worse than optimizing any policy instrument individually (Roberts and Spence 1976; Weitzman 1978). Second, analysis focusing on economic impacts, and abstracting from the political economy of implementation policy instruments, shows that no single policy instrument clearly dominates the others (Fullerton 2001; Nordhaus 2007; Goulder and Parry 2008). Third, the regulatory policy needs to influence multiple decisions, in our case, the capital investment in innovation, and the decision about the

[1]There are also reasons against hybrid cap-and-trade markets, such as the increased design complexity, commitment problems, political influence activities (e.g., price controls are subject to significant lobbying activities, as are already the number of emissions permits as well as the mode of the initial permit allocation), and increased risk of incompatibilities of different designs across countries (cf. Section 4).

emissions output. In terms of results, we characterize the optimal policy, a cap-and-trade scheme with price controls (in the form of price cap and price floor), and contrast it to the classical schemes of pure taxation and cap-and-trade without price controls, with respect to performance and the firms' incentives to abate and innovate. Tight price controls limit the firms' exposure to risk on the carbon market due to macroeconomic risk and uncertainty resulting from the random return to the firms' R&D investment. However, they also lead to variability in aggregate emissions and thus increase the volatility of environmental damages. The latter because of their convexity in aggregate emissions lead (via Jensen's inequality) to higher expected damages. While the firms' innovation becomes more effective, it is always optimal to set a more ambitious, lower emissions cap, optimal price controls tighten only when the slope of the marginal environmental damage cost curve (per unit of emissions output) is small. When the slope of marginal environmental damages is large, then it is best for a regulator to relax price controls in response to an increase in the firms' innovation effectiveness. While at first glance this appears to be a contradiction to Weitzman's (1974) seminal analysis of 'prices vs. quantities,' which predicts – in the absence of innovation – less stringent price controls as a consequence of an increase in marginal environmental damages, the same tradeoff prevails at any fixed level of innovation effectiveness. This, together with the fact that price controls are relaxed (in the sense that price floor decreases and price cap increases) when innovation effectiveness increases and environmental damages are large, implies that the sensitivity of the price controls increases substantially as innovation becomes an important factor. This increased sensitivity underlines the necessity to jointly consider all available instruments, as welfare losses are compounded in the presence of additional uncertainty, e.g., in terms of macroeconomic conditions, innovation costs, or environmental damages.

1.1 Related Literature

Ever since Tyndall's (1861) empirical investigations about the interaction of gases with radiation and concomitant absorption of heat, and Arrhenius' (1898) theoretical model of the greenhouse effect, the question of global warming, in terms of its causes, description, mitigation of its effects, and projection of resulting scenarios, has been on the modern research agenda. A consensus emerged that carbon emissions by man's economic activity and climate change are intertwined, and have to be addressed simultaneously (see, e.g., Nordhaus (1977)). The economic activity considered here includes two decisions taken by firms: first, how much carbon (dioxide) to emit ('emissions control'), and, second, how much to invest in an improvement ('innovation') of carbon-abatement technologies.

Emissions Control. Pigouvian taxation (Pigou 1920) was initially viewed as the most straightforward way to price the social cost of firms' emissions output, since unlike other distortionary taxation on a firm's inputs, which usually lead to significant deadweight losses (Ballard et al. 1985), a carbon tax corrects a distortion generated by the lack of a price for the expected environmental damages through carbon (or 'carbon-equivalent') emissions (Pearce 1991). An alternative course of action for governments, namely to issue tradable emissions permits, was suggested already by Coase (1960) and further developed by Crocker (1966), Dales (1968), and Montgomery (1972). Such quantity-based regulation is sometimes viewed as inferior on the grounds of significant transaction costs (Stavins 1995), given that an administrative system for levying tax is usually available. In the absence of transaction-cost considerations, the optimal choice between tax or quantity-based allowance-trading scheme depends on the nature of the uncertainty (Weitzman 1974):[2] since environmental damages are typically modelled as a convex function of the aggregate emissions output, an increase in risk (Rothchild and Stiglitz 1971) increases expected damages (as a consequence of Jensen's inequality), which in turn favors quantity-based regulation. If, on the other hand, the loss in society's payoffs due to uncertainty in emissions output and resulting expected environmental damages are small compared to the loss due to randomness in market prices, then an emissions tax is preferred, as it eliminates price uncertainty. Naturally, as Weitzman (1978) shows, a combination of price and quantity regulation cannot do worse than either policy instrument alone. In Weitzman's treatment a price-quota system determines a socially optimal reward as a *function* of its emissions output for each participating firm. In actual real-world settings, it is impossible to implement such infinite-dimensional policies (in the form of reward functions) using simple cap-and-trade. Yet, a first approximation, which still combines the features of pure taxation and a simple cap-and-trade system, is a market for emissions allowances with price controls: an emissions cap controls total emissions and determines the initial num-

[2] Various generalizations of Weitzman's (1974) analysis have been proposed, such as for situations with asymmetric information between regulator and firms (Laffont 1977; Hoel and Karp 2002), correlated uncertainty (Stavins 1996), hybrid price-quantity controls (Roberts and Spence 1976; Weitzman 1978), incomplete enforcement (Montero 2002), and bankable permits (Fell et al. 2008).

ber of permits to be issued. As long as the resulting market price for emissions is within pre-specified price bounds, this results in a normal cap-and-trade system. If the carbon price reaches the pre-specified maximum price (price cap), additional permits are issued (Roberts and Spence 1976; McKibbin and Wilcoxen 2002; Pizer 2002). The price floor can be implemented with a reservation price for allowance auctions (Hepburn et al. 2006), by governments issuing option contracts on the carbon price (Ismer and Neuhoff 2006) as a commitment to buy back permits as the carbon price drops below the price floor. More recently, Philibert (2008) uses extensive Monte-Carlo simulations to study the effect of price caps and price floors on climate policy. The simulation results confirm that price controls, while increasing expected environmental damage, dampen expected aggregate abatement cost. To this we add a simplified formal framework and an analysis of the interaction with innovation.

Technological Innovation. The introduction of a carbon price via incentive-based emissions-control policies is, according to Hicks' (1932, p. 132) 'induced invention hypothesis,' likely to affect the rate and path of technological change. This hypothesis sparked not only a stream of research attempting to formally establish this effect in general-equilibrium models (Kennedy 1964; von Weiszäcker 1965), but also several severe criticisms related to the description of knowledge accumulation (Samuelson 1965; Nordhaus 1973), producing a hiatus of results. More recently, research on technology-induced innovation has seen renewed efforts, particularly in the context of carbon-abatement technologies (Goulder and Mathai 2000; Goulder and Schneider 1999; Sue Wing 2003). For a recent survey on technological change in economic models of environmental policy, see Löschel (2002) and Edenhofer et al. (2006), who emphasize the need to consider innovation as an endogenous decision variable rather than an exogenous process. With the aid of simulations in a computational general-equilibrium model, Goulder and Mathai (1999) show that a carbon tax may stimulate R&D and technological progress in both carbon-using and carbon-competing industries. Goulder and Mathai (2000) demonstrate analytically that induced technological change leads to a lower carbon tax. Our findings, which are obtained in a somewhat different setting, where firms can fully appropriate rents to innovation, only partially confirm this finding. When the intensity (or 'effectiveness') of innovation is large, then in our model a decrease in tax is an optimal response.[3] Yet, for relatively small induced technological change, which corresponds to fairly realistic model calibration, it may be optimal to increase taxes as a response to an increase in innovation effectiveness. More generally, in a hybrid cap-and-trade scheme with quantity controls, an increase in innovation effectiveness can lead to either a tightening of the price band (for low marginal environmental damages) or a widening of the price band (for high marginal environmental damages). Margolis and Kammen (1999) point out that investments in R&D in the U.S. energy sector are low when compared to other sectors, somewhat corroborating the possibility that innovation effectiveness may still be low for carbon-abatement technologies.

1.2 Outline

The paper is organized as follows. After introducing the basic model in Section 2, we characterize, in Section 3, optimal regulatory approaches and their respective responses to increases in innovation effectiveness. Policy implications are discussed in Section 4, and Section 5 concludes.

2 The Model

We consider a unit mass of firms, indexed by $\theta \in \Theta \subset \mathbb{R}_{++}$, and distributed on the (measurable) type space Θ with the cumulative distribution function (cdf) $F : \Theta \to [0, 1]$, so that the mean

$$\mu = \int_\Theta \theta dF(\theta)$$

and the variance

$$\sigma_\theta^2 = \int_\Theta (\theta - \mu)^2 dF(\theta)$$

both exist and are finite. The model timing consists of three time periods (stages), indexed by $t \in \{0, 1, 2\}$. At time $t = 0$ (*regulation stage*), a regulator commits to a regulatory policy $R = (E, L, U)$, by announcing an emissions cap E, a price floor L, and a price cap U. In particular, the regulator may choose pure taxation or a cap-and-trade scheme

[3] Hart (2008) examines the intertemporal use of carbon taxes in the presence of technological spillovers. He shows that a regulator may find it optimal to set taxes above the Pigouvian level of marginal environmental damages in order to provide a sufficiently large incentive to innovate.

without price controls.[4] At time $t = 1$ (*innovation stage*), any firm has the option to invest in innovation, which may reduce its cost of carbon abatement in the future. Finally, at time $t = 2$ (*implementation stage*), each firm $\theta \in \Theta$ chooses its carbon emissions level. We now describe each of the three stages in detail, starting with the last.

Implementation Stage ($t = 2$). Without any price on carbon or other output restrictions, firm θ expects to produce its business-as-usual (BAU) carbon emissions output of $e_0(\theta)$. The actual BAU carbon emissions for firm θ are subject to a macroeconomic random shock $\tilde{\varepsilon}$. The latter is common to all firms and has cdf $G : \mathbb{R} \to [0, 1]$, with $E\tilde{\varepsilon} = 0$ and $0 < E\tilde{\varepsilon}^2 = \sigma_\varepsilon^2 < \infty$.[5] Given a realized BAU carbon emissions level $\hat{e}_0 = e_0(\theta) + \varepsilon$, firm θ's cost of abating its carbon emissions to a level $e \leq \hat{e}_0$ is

$$C(e, \hat{\rho}\theta|\hat{e}_0) = \frac{(\hat{e}_0 - e)^2}{2\hat{\rho}\theta}, \tag{1}$$

where $\hat{\rho} \geq 1$ denotes the outcome of the firm's investment in the preceding innovation stage, further detailed below. All else being equal, the larger the firm's type θ, the smaller its marginal abatement cost $(\hat{e}_0 - e)/(\hat{\rho}\theta)$.[6] Given a price p for each unit of carbon emissions (typically measured in tCO_2eq, i.e., tons of 'carbon-dioxide equivalent,'[7] and here denoted by tCO_2 for simplicity), the firm's total cost of producing at the emissions

[4]In Section 3, these important special cases are examined separately.

[5]It is possible to allow for independent firm-specific (idiosyncratic) zero-mean random shocks $\tilde{\varepsilon}(\theta)$ with finite variances $\sigma_\varepsilon^2(\theta) > 0$, satisfying the Lindeberg condition $\int_\Theta E\left[\tilde{\varepsilon}^2(\theta)/\sigma_\varepsilon^2(\theta)\big|\, |\tilde{\varepsilon}(\theta)| > \delta\sigma_\varepsilon(\theta)\right] dF(\theta) = 0$ for all $\delta > 0$, which complicates the presentation and leads to an equivalent result. By the central limit theorem, the macroeconomic shock $\tilde{\varepsilon}$ then corresponds to the limiting zero-mean normal distribution with standard deviation $\sigma_\varepsilon = \int_\Theta \sigma_\varepsilon(\theta) dF(\theta)$.

[6]The affine form of the marginal abatement cost is chosen as in Weitzman (1978) so as to obtain explicit model results. More generally, marginal abatement costs are convex and decrease to zero as the emissions level approaches the firm's BAU level (Misfeldt and Hauff 2004). Enkvist et al. (2007) use data to find an approximately affine marginal abatement-cost curve. They also note that marginal abatement cost may become negative for small levels of abatement, as small emissions improvements could be implemented at a gain to a firm. In our model we assume that all such gains have been internalized, so that the marginal abatement cost at the firm's realized BAU emissions level vanishes.

[7]One ton of *carbon dioxide equivalent* (denoted by tCO_2eq) is the weight of a greenhouse gas which would have the same time-integrated radiative forcing (over a period of 100 years) as one ton of CO_2. The term *radiative forcing*, as used by the IPCC, refers to the perturbation of the surface-troposphere system after introduction of a chemical agent, e.g., as a result of a change in greenhouse-gas concentrations. A related measure, *equivalent carbon dioxide* (CO_2e), is the concentration of CO_2 that would generate the same level of radiative forcing as a given type of greenhouse gas. The unit of CO_2e is parts per million per

level e is

$$TC(e, p, \hat{\rho}\theta|\hat{e}_0) = C(e, \hat{\rho}\theta|\hat{e}_0) + pe.$$

The firm's optimal emissions output minimizes its total cost, and is uniquely determined by[8]

$$e^*(p, \hat{\rho}\theta|\hat{e}_0) = \hat{e}_0 - \hat{\rho}\theta p. \tag{2}$$

At this output level, the firm's optimal total carbon emissions cost is

$$TC^*(p, \hat{\rho}\theta|\hat{e}_0) = \hat{e}_0 p - \frac{\hat{\rho}\theta p^2}{2}.$$

Innovation Stage ($t = 1$). Given the announcement of a carbon pricing policy, each firm θ chooses the level y of innovative activity. The cost of pursuing the innovative activity $K(y)$ is known, but the outcome of the innovation $\tilde{\rho}(y)$ is uncertain. The innovation provides an advantage over the existing technology if and only if $\tilde{\rho}(y) > 1$. Only in this case will it be utilized. We assume that the expected outcome of innovation for a certain level of innovative activity y is:[9]

$$y = \mathbb{E}\left[\max\{\tilde{\rho}(y), 1\}|\, y\right] - 1 \geq 0. \tag{3}$$

Firm θ's expected net payoff from innovating is

$$\pi(p, y, \theta) = \frac{\theta y p^2}{2} - K(y).$$

Assuming that $K(y)$ is a continuously differentiable, convex, and increasing function (satisfying the Inada conditions $K(0) = K'(0) = 0$ and $K'(\infty) = \infty$), the optimal innovation is determined by the first-order optimality condition $\theta p^2/2 = K'(y)$. If we assume, for the sake of discussion, that

$$K(y) = cy^2/2,$$

volume (ppmv). CO_2eq is therefore a time-integrated version of CO_2e and measures the 'global warming potential' of a given amount of greenhouse gas emissions.

[8]In principle it is possible to obtain negative values for optimal carbon emissions, which implies that the firm would further substitute its production away from carbon than its zero-carbon emissions normalization would indicate. Alternatively, the firm can accumulate carbon credits. The unconstrained optimization also simplifies the model in that the expected level of aggregate emissions with macroeconomic uncertainty corresponds to the aggregate emissions level in the absence of this uncertainty ('certainty equivalence'). Relaxing this condition would influence modeling results only marginally, and would also raise the additional question of the precise measurement of the absolute level of BAU emissions.

[9]This is without any loss in generality, as for any arbitrary parametrization of the innovative process $\tilde{\rho}(x)$ in terms of x, one can simply set y equal to $v(x) \equiv \mathbb{E}\left[\max\{\tilde{\rho}(x), 1\}|\, x\right] - 1$ and then reparameterize the innovative process in terms of $\hat{x} = (\hat{x}_0, x)$ which contains $\hat{x}_0 = y = v(x)$ as one component.

where c is a positive constant, firm θ's optimal innovation becomes

$$y^*(p,\theta) = \frac{\theta p^2}{2c}, \tag{4}$$

resulting in an expected net payoff of

$$\pi^*(p,\theta) = \frac{\theta^2 p^4}{8c}. \tag{5}$$

Thus, the benefits of improving abatement technologies are highly sensitive to the carbon price. Nonetheless, the expected payoff of optimal innovation is positive, as long as small improvements are cheap (since $K'(0) = 0$).[10]

Regulation Stage ($t = 0$). The regulator commits to a (deterministic) regulatory policy

$$R = (E, L, U),$$

consisting of an emissions cap E (implemented by issuing a set quantity of emissions permits), and a price interval $[L, U]$ for the secondary market in emissions permits. In the event the market price p reaches the price floor L, the regulator offers firms to buy back emissions permits at the price L. If the market price p reaches the price cap U, the regulator offers firms additional permits at the price U.

Remark 1 (i) A pure carbon tax τ can be implemented by choosing $R = (E, \tau, \tau)$, where $E \geq 0$ is arbitrary, since the carbon market is bypassed by the regulator, who offers an ex-ante unlimited number of carbon emissions permits at the fixed price of τ. (ii) A pure carbon emissions cap of E is also a special case, which can be implemented by setting $R = (E, 0, \infty)$, effectively disabling the price controls with $L = 0$ and $U = \infty$.

The set of feasible regulatory policies is

$$\mathcal{R} = \left\{ (E, L, U) \in \mathbb{R}^3_+ : L \leq U \right\}.$$

To formulate the regulator's problem, we first aggregate the firms' expected emissions at a given carbon price p, which yields (using Eqs. (2)–(4)) the expected aggregate carbon emissions output

$$Q(p,\varepsilon) = \int_\Theta e^*(p, (1 + y^*(p,\theta))\theta | e_0(\theta) + \varepsilon) dF(\theta) = e_0 + \varepsilon - \mu p - \frac{\mu^2 + \sigma_\theta^2}{2c} p^3, \tag{6}$$

[10]Enkvist et al. (2007) argue that the cost of abating the first units of carbon emissions, net of benefits, may on average be negative. The assumption that $K'(0) = 0$ implies that any firm's BAU emissions are set to the level at which it has internalized any such abatement benefits.

where we denote by

$$e_0 = \int_\Theta e_0(\theta) dF(\theta)$$

the expected aggregate BAU emissions output in the economy. To understand the integral in Eq. (6) note first that by Eq. (3) a firm of cost type θ, when investing in technology at the optimal innovation level $y^*(p,\theta)$, expects to transition to the improved cost type $\hat{\theta} = (1 + y^*(p,\theta))\theta$. After substitution of Eqs. (2) and (4) in the integrand on the right-hand side of Eq. (6) is obtained via straightforward integration . At the aggregate emissions level Q, environmental damages are given by

$$D(Q) = \frac{dQ^2}{2},$$

where d denotes the slope of the marginal environmental damage cost curve.[11] The expected environmental damages are therefore

$$\bar{D}(R) = \mathbb{E}\left[D\left(Q(\tilde{p},\tilde{\varepsilon})\right) \middle| H(\tilde{p},\tilde{\varepsilon},R) = 0 \right], \tag{7}$$

where the measure of the stochastic price $\tilde{p}$ is determined by the market-clearing condition [12]

$$H(p,\varepsilon,R) \equiv (U - p)(p - L)(E - Q(p,\varepsilon)) = 0. \tag{8}$$

Insofar as the regulatory policy R influences aggregate emissions, it also controls the level of expected environmental damages. The expected aggregate cost of carbon abatement at the policy R is

$$\bar{C}(R) = \mathbb{E}\left[C\left(e^*(\tilde{p},(1 + y^*(\tilde{p},\tilde{\theta}))\tilde{\theta}|e_0 + \tilde{\varepsilon}),(1 + y^*(\tilde{p},\tilde{\theta}))\tilde{\theta} \middle| e_0 + \tilde{\varepsilon} \right) \middle| H(\tilde{p},\tilde{\varepsilon},R) = 0 \right], \tag{9}$$

where the optimal emissions e^* are given in Eq. (2) and optimal innovation is determined by Eq. (4). The firms' expected aggregate social cost of innovation $\bar{K}(R)$ of the policy R is

$$\bar{K}(R) = \lambda \mathbb{E}\left[K(y^*(\tilde{p},\tilde{\theta})) \middle| H(\tilde{p},\tilde{\varepsilon},R) = 0 \right], \tag{10}$$

where the constant $\lambda \in [0,1]$ describes how much society (i.e., the regulator) cares about these costs. The reason why one would expect generally that $\lambda < 1$ is that firms are

[11] A quadratic form for environmental damages is widely used in the literature (see, e.g., Baumol and Oates (1988)). Such quadratic damages seem more realistic, especially in light of nonlinear threshold effects for high CO_2 concentrations, than the often assumed linear form (see, e.g., Tol 2005).

[12] For the measure of $\tilde{p}$ to be determined by the condition $H(\tilde{p},\tilde{\varepsilon},R) = 0$ it is, by the inverse function theorem, enough to assume that $\partial H(p,\varepsilon,R)/\partial p$ exists and is nonzero almost everywhere.

able to appropriate a portion of the innovation payoffs in the form of intellectual property rights (on top of the abatement-cost savings), resulting in private benefits, e.g., through international technology licensing or savings in future unmodelled periods, that offset the innovation cost to society at least in part.[13]

The Regulatory Problem. An *optimal regulatory policy* R^* maximizes expected *welfare* $\bar{W}(R)$ (or, equivalently, minimizes total expected *social cost* $\overline{SC}(R) \equiv -\bar{W}(R)$),

$$\bar{W}(R) = -\bar{C}(R) - \bar{D}(R) - \bar{K}(R), \tag{11}$$

i.e., it is such that

$$R^* \in \arg\max_{R \in \mathcal{R}} \bar{W}(R).$$

The main notation relevant for the model is summarized in Table 1.

3 Optimal Regulation

Common regulatory schemes include pure carbon taxation, cap-and-trade markets without price controls, and cap-and-trade markets with price controls as a generalization which includes the first two. Here we examine all three regulatory schemes. In doing this, we parameterize the firms' 'innovation effectiveness' by $\beta \geq 0$ and examine the effect of innovation as β increases, in particular over the 'base case' without innovation (when $\beta = 0$). Such increases could come about exogenously as a result of government sponsorship, or endogenously through 'learning by doing' with private investment in R&D, which tends to generate further technological possibilities (Grubb 1997).

To compute the regulator's objective function (expected total welfare $\bar{W}$), we first determine the price $p(\varepsilon, R)$ of carbon using the market-clearing condition (8) as a function of the macroeconomic shock ε and the regulatory policy $R = (E, L, U)$, which yields

$$p(\varepsilon, R) = \begin{cases} U, & \text{if } \varepsilon \geq \bar{\varepsilon}(E, U), \\ L, & \text{if } \varepsilon \leq \underline{\varepsilon}(E, L), \\ p_0(\varepsilon, E) - \Delta(p_0(\varepsilon, E), \beta), & \text{otherwise,} \end{cases} \tag{12}$$

[13] In addition, society may be able to obtain a "double dividend" from the revenues generated by the sale of the carbon permits (Bovenberg and de Mooij 1994; Carraro et al. 1996). For simplicity, we assume that the double dividend is zero.

Symbol	*Description*	*Unit*
c	Marginal cost of innovation	\$
d	Slope of marginal environmental damage cost	$\$/(tCO_2)^2$
$e^*(p,\theta)$	Firm θ's carbon emissions output at price p	tCO_2
e_0	Aggregate BAU emissions output	tCO_2
p, p_0	Carbon price, with and without innovation	$\$/(tCO_2)$
$y^*(p,\theta)$	Firm θ's innovative activity at price p	N/A
$\bar{C}(R)$	Expected aggregate abatement cost	\$
$\bar{D}(R)$	Expected aggregate environmental damage cost	\$
$\bar{K}(R)$	Expected social cost of innovation	\$
E	Emissions cap	tCO_2
L	Price floor	$\$/(tCO_2)$
$Q(p,\varepsilon)$	Aggregate carbon emissions level at (p,ε)	tCO_2
R	Regulatory policy, $R=(E,L,U)\in\mathcal{R}\subset\mathbb{R}^3_+$	$[(E,L,U)]$
U	Price cap	$\$/(tCO_2)$
$\bar{W}(R)$	Expected aggregate social welfare	\$
β	Innovation effectiveness	$(tCO_2)^2/\$^2$
ε	Macroeconomic uncertainty	tCO_2
λ	Regulator's weight on firms' aggregate profit in $\bar{W}$	N/A
θ	Cost type in type space $\Theta\subset\mathbb{R}_{++}$	$(tCO_2)^2/\$$
τ	Carbon tax	$\$/(tCO_2)$

Table 1: Summary of Notation.

where

$$p_0(\varepsilon, E) = (e_0 + \varepsilon - E)/\mu \tag{13}$$

is the market price for carbon without innovation, and

$$\begin{aligned}
\bar{\varepsilon}(E, U) &= \mu U(1 + \beta U^2) - (e_0 - E), && (14)\\
\underline{\varepsilon}(E, L) &= \mu L(1 + \beta L^2) - (e_0 - E) && (15)
\end{aligned}$$

are the upper and lower thresholds for BAU emissions realizations that trigger price controls. The perturbation term in Eq. (12),

$$\Delta(p_0, \beta) = 2\mu A^{-1/3} - \frac{A^{1/3}}{6\beta} + p_0, \tag{16}$$

with $A = 12\mu\beta^2 \left(9p_0 + \sqrt{3}\sqrt{(4\mu^3/\beta) + 27p_0^2}\right)$, is nonnegative and increasing in the *innovation effectiveness*

$$\beta = \frac{\mu^2 + \sigma_\theta^2}{2\mu c}. \tag{17}$$

The innovation effectiveness increases when the innovation cost c decreases, or when the first (μ) or second (σ_θ^2) moment of the firms' type distribution F increases. When innovation becomes prohibitively expensive (so that $c \to \infty$), then β vanishes and consequently $\Delta(p_0, 0) = 0$. The expected social welfare $\bar{W}$ as a function of R in Eq. (11) is obtained by substituting the market price $p(\varepsilon, R)$ in Eqs. (7), (9), and (10), leading to

$$\begin{aligned}
\bar{C}(R) &= \frac{\mu}{2}\int_{-\infty}^{\infty} \left(1 + \beta p^2(\varepsilon, R)\right) p^2(\varepsilon, R)\, dG(\varepsilon), && (18)\\
\bar{D}(R) &= \frac{d}{2}\int_{-\infty}^{\infty} \left(e_0 + \varepsilon - \mu p(\varepsilon, R)\left(1 + \beta p^2(\varepsilon, R)\right)\right)^2 dG(\varepsilon), && (19)\\
\bar{K}(R) &= \frac{\lambda\mu}{4}\int_{-\infty}^{\infty} \beta p^4(\varepsilon, R) dG(\varepsilon). && (20)
\end{aligned}$$

3.1 Pure Taxation

Consider first a restriction of feasible policies to imposing a carbon tax $\tau \geq 0$, which is equivalent to fixing a price in a carbon market by setting $L = U = \tau$ and issuing an arbitrary number of permits, e.g., $E = e_0$. In that case, $R = (e_0, \tau, \tau)$ and $p(\varepsilon, R) \equiv \tau$.

Proposition 1 (Optimal Tax) *For any $\beta \geq 0$, let $\tau^*(\beta)$ be the optimal carbon tax.*

(i) In the absence of innovation, i.e., when $\beta = 0$, the optimal tax is

$$\tau^*(0) = \frac{de_0}{1 + \mu d} \equiv \tau_0^*.$$

(ii) The optimal tax $\tau^*(\beta)$ decreases (in a neighborhood of $\beta > 0$) if and only if

$$\mu d > (1-\lambda)/(1+3\beta\tau^2)^2\big|_{\tau=\tau^*(\beta)} .$$

(iii) For $\beta \to \infty$, the optimal tax vanishes, i.e., it is $\tau^*(\infty) = 0$.

Part (i) implies that the optimal social welfare without innovation is

$$\bar{W}^*_{0,\mathrm{Tax}} = -\frac{\mu d e_0^2}{1+\mu d} - \frac{d\sigma_\varepsilon^2}{2}.$$

The maximum tax $\tau^*_{\max} = \max_{\beta\geq 0} \tau^*(\beta)$ follows directly from part (ii) when the inequality is either replaced by an equality or satisfied everywhere (in which case τ_0^* is maximal), so that

$$\tau^*_{\max} = \begin{cases} \tau_0^*, & \text{if } \mu d \geq 1-\lambda, \\ \sqrt{\frac{1}{3\beta_m}\left[\sqrt{\frac{1-\lambda}{\mu d}} - 1\right]_+}, & \text{otherwise;} \end{cases}$$

it is imposed where $\beta = \beta_m \equiv \inf\left\{\hat{\beta} \geq 0 : \mu d \geq (1-\lambda)/(1+3\hat{\beta}\tau^2)^2\Big|_{\tau=\tau^*(\hat{\beta})}\right\}$. The fact that the optimal carbon tax is (as long as $\lambda < 1$) generally nonmonotonic in innovation effectiveness, as shown in Figure 1, is noteworthy. Indeed, if environmental damages are 'small,' so that $\mu d < 1-\lambda$, then it is optimal for a regulator to first increase taxes as innovation starts to become feasible (i.e., for small β). The intuition is that higher taxes lead to increased incentives to innovate and thus imply more ambitious emissions targets. The regulator is more likely to increase taxes as a response to increases in β, the larger $1/\mu$ (corresponding, roughly, to the average marginal abatement cost) and the larger the weight on the firms' aggregate profits in the expected social welfare. As innovation becomes more and more effective, the regulator can decrease taxes, since firms will tend to fully abate their emissions, even when the carbon tax is low. If the regulator considers all of the innovation cost as social cost, so that $\lambda = 1$, the optimal tax is decreasing in the innovation effectiveness β.

Example 1 Consider an economy where the firms' marginal cost types are distributed such that $\mu = 33 \cdot 10^6\,(t\mathrm{CO}_2)^2/\$^2$, $\sigma_\theta = 3.3 \cdot 10^6\,(\mathrm{tCO}_2)^2/\$^2$, the innovation-cost coefficient is $c = \$100 \cdot 10^9/(\text{unit of relative improvement})$, and the environmental damage function is characterized by $d = \$3.3 \cdot 10^{-9}/(t\mathrm{CO}_2)^2$. Then for an annual aggregate BAU emissions level of $e_0 = 13.5 \cdot 10^9$ tCO_2, corresponding to the combined OECD emissions output (IEA 2008), we obtain that $\tau_0^* \approx \$40/(\mathrm{tCO}_2)$. Furthermore, with a

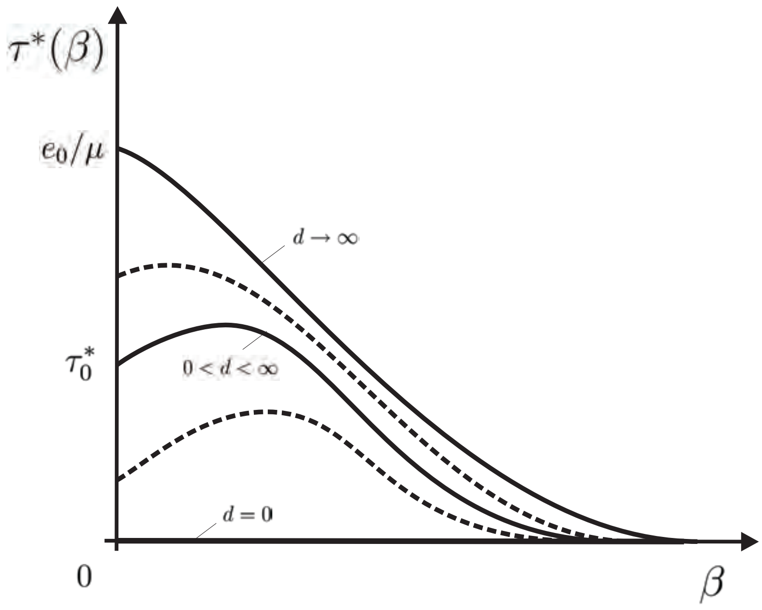

Figure 1: Optimal Carbon Tax as a Function of β and d (for $0 \leq \lambda < 1$).

resulting innovation effectiveness of $\beta \approx 0.167 \cdot 10^{-3}\,(\text{tCO}_2)^2/\$^2$, the optimal carbon tax (determined numerically) is $\tau^*(\beta) \approx \$46/(\text{tCO}_2)$ increases by about 13% over its level without innovation, for $\lambda = 0$. If the regulator considers the innovation cost as social expenses, i.e., when $\lambda = 1$, the optimal carbon tax drops to $\tau^*(\beta) \approx \$39/(\text{tCO}_2)$. The resulting expected aggregate emissions are $\bar{Q}_0^* \approx 12.17 \cdot 10^9$ tCO_2 without innovation, and $\bar{Q}^*(\beta) \approx 12.0 \cdot 10^9$ tCO_2 for $\lambda = 0$ (resp. $\bar{Q}^*(\beta) \approx 12.21 \cdot 10^9$ tCO_2 for $\lambda = 1$) with innovation. $\square$

3.2 Basic Cap-and-Trade (without Price Controls)

We now consider the special case where the regulator chooses a "basic" cap-and-trade scheme without binding price bounds, by setting $L = 0$ and $U = \infty$. Then the market price for carbon depends only on the emissions cap E and the realization of the macroeconomic uncertainty ε, i.e., $p = p(\varepsilon, E) = p_0(\varepsilon, E) - \Delta(p_0(\varepsilon, E), \beta)$. It is determined by the market-clearing condition (8), which can be written in the form

$$p_0(\varepsilon, E) - p(\varepsilon, E) - \beta p^3(\varepsilon, E) = 0. \tag{21}$$

Proposition 2 (Optimal Emissions Cap) *For any $\beta \geq 0$, let $E^*(\beta)$ be the optimal emissions cap in the absence of price controls.*

(i) In the absence of innovation, i.e., when $\beta = 0$, the optimal emissions cap is

$$E^*(0) = \frac{e_0}{1+\mu d} \equiv E_0^*.$$

(ii) As the firms' innovation effectiveness β increases, the optimal emissions cap $E^*(\beta)$ decreases.

(iii) For $\beta \to \infty$, the optimal emissions cap vanishes, i.e., $E^*(\infty) = 0$.

In the absence of innovation, Eq. (13) and part (i) of Propositon 1 imply that the expected price is the same as the optimal tax without innovation computed in the previous section,

$$\bar{p}_0^* = \mathbb{E}\left[p_0(\tilde{\varepsilon}, E_0^*)|E_0^*\right] = (e_0 - E_0^*)/\mu = \tau_0^*. \tag{22}$$

The corresponding optimal expected social welfare is

$$\bar{W}^*_{0,\text{Basic C\&T}} = -\frac{\mu d e_0^2}{1+\mu d} - \frac{\sigma_\varepsilon^2}{2\mu}.$$

Part (ii) of Proposition 2 characterizes the behavior of the solution in β. Since this paper provides several such monotone comparative statics results, we provide the proof intuition. The proofs of other such results (cf. Proposition 1 (ii) and Proposition 4 (ii),(iii)) follow along similar lines. Via implicit differentiation of the market-clearing condition (21), we obtain that $p_E, p_\beta < 0 < p_{\beta E}$ (see Eqs. (24)–(26) in the Appendix for the exact expressions), where subscripts denote partial derivatives. That is, when the carbon market clears, the market price p decreases in the emissions cap E and innovation effectiveness β; it also exhibits increasing differences in (β, E), which means that the price decrease in E (resp. β) is moderated when β (resp. E) increases. Furthermore, since p is nonnegative and decreasing in both β and E, any (positive integer) power of p also has increasing differences in (β, E). Using the previous relations, expected abatement cost $\bar{C}$ can easily be shown to have increasing differences in (β, E), while $\bar{D}$ does not depend on β for a given emissions cap. Thus, the expected social welfare $\bar{W}$ has decreasing differences in (β, E), so that the optimal E is decreasing in β. In other words, as innovation becomes easier (β increases), it is optimal to impose a more ambitious emissions cap, i.e., $E^*(\beta)$ is decreasing in β (cf. Figure 2).

Example 2 Using the same values for μ, σ_θ, c, d, and e_0 as in Example 1, we obtain an optimal emissions cap of $E_0^* \approx 12.2 \cdot 10^9$ tCO_2. Since $\mu d \approx 0.11 < 1$, a carbon tax is

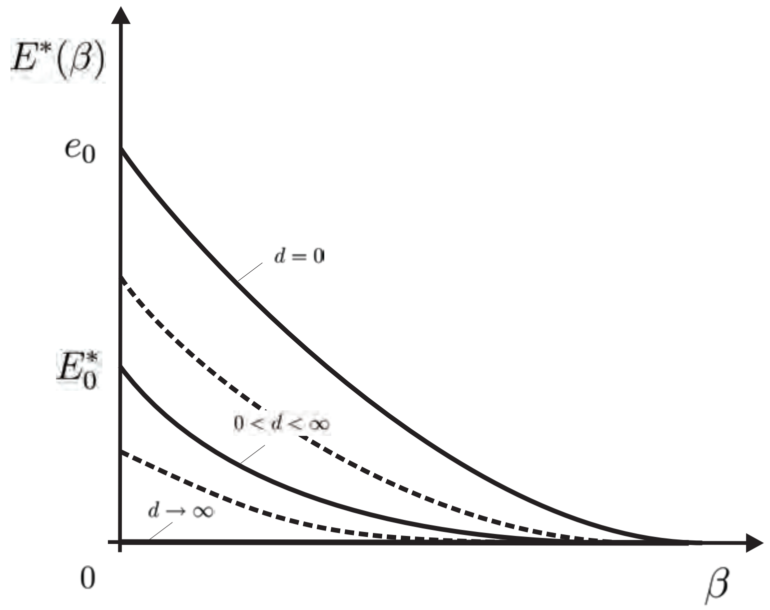

Figure 2: Optimal Emissions Cap as a Function of β and d.

superior to a pure emissions cap in terms of expected welfare. Furthermore, with an innovation effectiveness of $\beta \approx 0.167 \cdot 10^{-3}\,(\mathrm{tCO_2})^2/\$^2$, and a uniform distribution of the macroeconomic uncertainty on $[-\delta, \delta]$ (where $\delta = (10\%) \cdot e_0$), the optimal emissions cap $E^*(\beta) \approx 11.38 \cdot 10^9$ $\mathrm{tCO_2}$ for $\lambda = 0$ (resp. $E^*(\beta) \approx 11.78 \cdot 10^9$ $\mathrm{tCO_2}$ for $\lambda = 1$) with innovation is more ambitious than the optimal emissions cap E_0^* without innovation. $\square$

Comparison between Pure Taxation and Basic Cap-and-Trade

The price fluctuations have a detrimental impact on social welfare compared to pure taxation if environmental damages are small, i.e., when $\mu d < 1$. Indeed, as shown by Weitzman (1974), the difference in optimal expected welfare levels,

$$\bar{W}^*_{0,\text{Tax}} - \bar{W}^*_{0,\text{Basic C\&T}} = (1 - \mu d)\,\frac{\sigma_\varepsilon^2}{2\mu},$$

favors a carbon tax over the basic cap-and-trade scheme if and only if $\mu d < 1$, i.e., if and only if the slope of marginal environmental damages is small compared to the (average) marginal abatement cost.[14] With the introduction of innovation, i.e., when $\beta > 0$ and small enough, the scales gradually tip towards quantity-based regulation, no matter what the environmental damages. Let $\bar{W}^*_{\text{Tax}}(\beta)$ and $\bar{W}^*_{\text{Basic C\&T}}(\hat{\beta})$ be the optimal welfare under a pure tax and an emissions cap, respectively.

[14]Strictly speaking, the expected marginal cost is $\int_\Theta (1/\theta) dF(\theta)$, which may vary somewhat from $1/\mu$, which is relevant in the comparison with the slope of marginal environmental damages d.

Proposition 3 (Pure Taxation vs. Basic Cap-and-Trade) *(i) An increase in β increases the relative attractiveness of quantity-based regulation over pure taxation, i.e., there exists a $\bar{\beta} > 0$ such that*

$$0 < \beta < \hat{\beta} < \bar{\beta} \quad \Rightarrow \quad \bar{W}^*_{\text{Basic C\&T}}(\beta) - \bar{W}^*_{\text{Tax}}(\beta) < \bar{W}^*_{\text{Basic C\&T}}(\hat{\beta}) - \bar{W}^*_{\text{Tax}}(\hat{\beta}).$$

(ii) For large levels of innovation effectiveness, i.e., as $\beta \to \infty$, quantity-based regulation strictly dominates pure taxation, so that $0 = \bar{W}^*_{\text{Basic C\&T}}(\infty) > \bar{W}^*_{\text{Tax}}(\infty) = -d\sigma^2_{\varepsilon}/2$.

With increasing innovation effectiveness abatement goes up and aggregate emissions tend to zero under either of the two regulatory schemes. The intuition for part (i) of Proposition 3 is that while expected social welfare is increasing under both regulatory policies, the increase is slower under pure taxation. With innovation firms perceive price uncertainty as positive, as the upside to a higher price is a disproportionately larger benefit from abating carbon and thus a higher expected return on innovation (the technical reason being that firms' payoffs are convex in the market price, which implies a preference for increases in risk). In the extreme, a fixed carbon tax does not respond to the macroeconomic uncertainty, which leads to residual emissions (or overabatement) and therefore to positive environmental damages, whereas quantity-regulation forces aggregate emissions to zero.

Remark 2 For any β, let $\bar{W}_{\text{Tax}}(E;\beta)$ and $\bar{W}_{\text{Basic C\&T}}(\tau;\beta)$ be the expected welfare under a pure-tax and a basic cap-and-trade scheme, respectively. The market-clearing condition (21) then implies that

$$\max_{E\geq 0} \bar{W}_{\text{Basic C\&T}}(E;\beta) = \max_{\tau\geq 0} \mathbb{E}\left[\bar{W}_{\text{Basic C\&T}}(e_0 + \tilde{\varepsilon} - \mu\tau\left(1+\beta\tau^2\right);\beta)\middle|\,\tau\right].$$

Similarly, the relation between price and aggregate emissions output in Eq. (6) yields that

$$\max_{\tau\geq 0} \bar{W}_{\text{Tax}}(\tau;\beta) = \max_{E\geq 0} \mathbb{E}\left[\bar{W}_{\text{Tax}}(p(\tilde{\varepsilon},E);\beta)\middle|\,E\right].$$

In other words, finding the optimal tax is equivalent to finding an emissions cap that maximizes the expected welfare subject to market clearing (21), and finding the optimal emissions cap is the same as optimizing the expected welfare subject to the output relation (6).

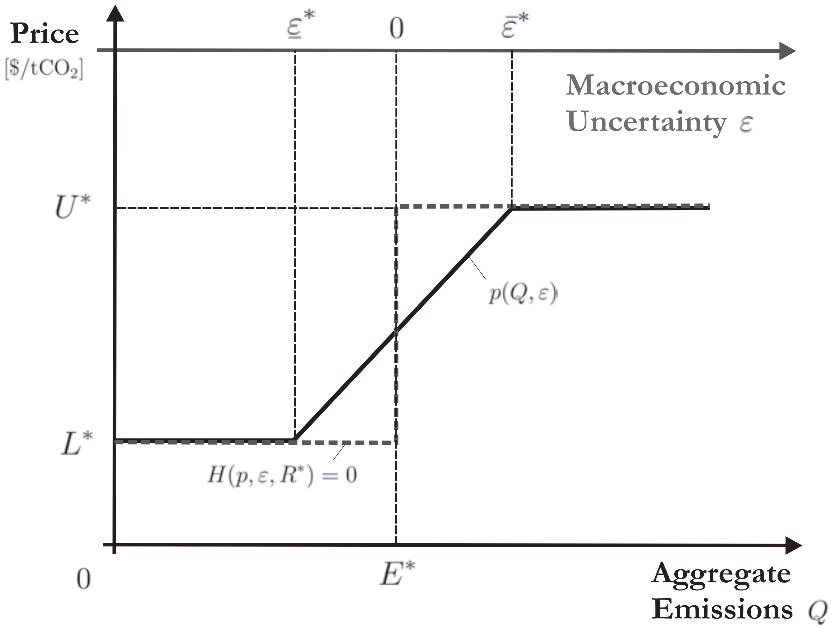

Figure 3: Cap-and-Trade with Price Controls.

3.3 Cap-and-Trade (with Price Controls)

Let us now consider the general case where the regulator can specify the general cap-and-trade scheme $R = (E, L, U)$ with price controls. Clearly, this scheme cannot perform worse than any of the two regulatory policies considered above. The extant theoretical literature has focussed on the effect of a price ceiling (McKibbin and Wilcoxen 2002; Pizer 2002), which by Eqs. (18) and (19) reduces expected aggregate costs, but at the same time tends to increase expected aggregate damages. The introduction of price ceilings therefore tends to convert abatement-cost uncertainty into environmental-damage uncertainty. As a result it may be optimal to increase or decrease the emissions cap, depending on how fast marginal environmental damages increase. A price floor, on the other hand, tends to increase the expected aggregate abatement cost and decrease expected aggregate damages, and therefore produces a counterveiling effect on the optimal emissions cap.

Proposition 4 (Optimal Cap-and-Trade with Price Controls) *(i) In the absence of innovation, i.e., when $\beta = 0$, the optimal regulatory policy $R^* = (E_0^*, L_0^*, U_0^*)$ is deter-*

mined by

$$E_0^* = \frac{e_0}{1+\mu d},$$
$$L_0^* = d \cdot \frac{e_0 + \mathbb{E}\left[\tilde{\varepsilon}|\tilde{\varepsilon} \leq \underline{\varepsilon}^*\right]}{1+\mu d},$$
$$U_0^* = d \cdot \frac{e_0 + \mathbb{E}\left[\tilde{\varepsilon}|\tilde{\varepsilon} \geq \bar{\varepsilon}^*\right]}{1+\mu d},$$

where the optimal uncertainty thresholds $\bar{\varepsilon}^* = \bar{\varepsilon}(E_0^*, U_0^*)$ and $\underline{\varepsilon}^* = \underline{\varepsilon}(E_0^*, L_0^*)$ are given by Eqs. (14) and (15), respectively. For parts (ii) and (iii), assume that the density of the macroeconomic uncertainty $\tilde{\varepsilon}$ is nondecreasing on its support.[15]

(ii) With increasing innovation effectiveness β, the optimal emissions cap $E^*(\beta)$ decreasses.

(iii) With increasing innovation effectiveness β, the optimal price controls $L^*(\beta)$ and $U^*(\beta)$ tighten for small d and loosen for large d.

The workings of an optimal cap-and-trade market design with price controls as a function of the macroeconomic uncertainty ε and aggregate emissions Q are illustrated in Figure 3. For $\beta = 0$ (i.e., without innovation), the optimal emissions cap E_0^* is unaffected by the optimal price controls and identical to the one determined earlier, in Section 3.2. The price controls are symmetric to the marginal environmental damages dE_0^* if the distribution of macroeconomic uncertainty is symmetrical. The width of the interval depends on thickness of the tails of that distribution and on magnitude of the product μd. The latter is also decisive in determining the tradeoff between pure taxation ($\mu d < 1$) and quantity-based regulation ($\mu d > 1$), as analyzed before. With increases in μd, not only does the regulator set a lower emissions cap, but also price controls are loosened around dE_0^*. Part (ii) of the last result states, that, all else being equal, innovation always leads to a more ambitious emissions target. In part (iii) of Proposition 4, it becomes evident that the evolution of the price controls as a function of innovation effectiveness is somewhat more complicated. First, the price cap and price always adjust to changes of β in opposite directions, either tightening or widening the interval of admissible prices in the market for carbon permits. Second, for small d price controls tighten with increasing β, i.e.,

[15]This condition is simple and sufficient; it is, e.g., for a uniform distribution of the macroeconomic uncertainty on a compact support. Much less is required, as can be seen by inspecting the proof of Proposition 4.

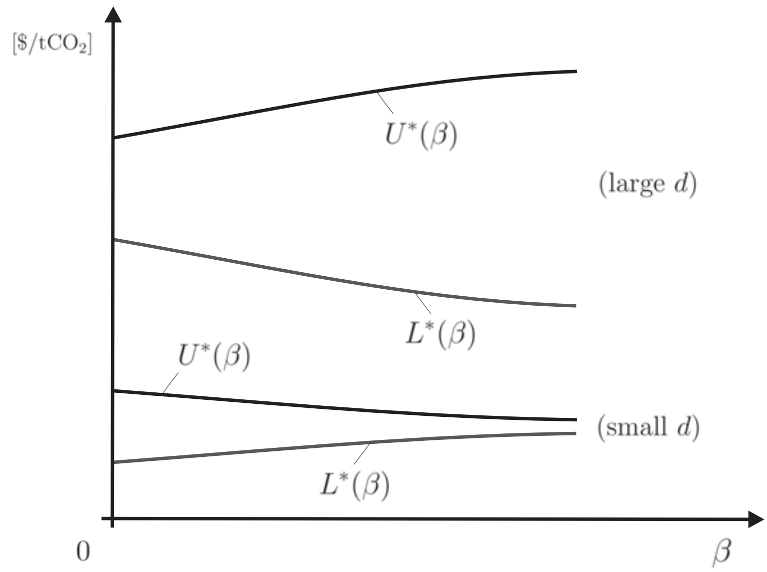

Figure 4: Price Controls as a Function of Innovation Effectiveness β (for d Small/Large).

$L^*(\beta)$ increases and $U^*(\beta)$ decreases. Third, for large d, it is best for the regulator to respond to an increase in β by relaxing price controls, so that $L^*(\beta)$ decreases and $U^*(\beta)$ increases. Combining the last two points, we see that the sensitivity of the optimal regulatory scheme to the magnitude of the slope of marginal environmental damages d increases with increasing β (cf. Figure 4). This makes sense and directly corresponds to the classical tradeoff by Weitzman. But this time it is related to the regulatory response to innovation. Depending on the magnitude of the environmental damages, an increase of innovation effectiveness may prompt a regulator to impose more or less price control, a decision which becomes more sensitive to the magnitude of marginal environmental damages.

Remark 3 Even for extremely large damage cost, the corresponding limits for the price bounds are well-defined,

$$\lim_{d\to\infty} U_0^* = \frac{e_0 + \mathbb{E}\left[\tilde{\varepsilon}|\tilde{\varepsilon} \geq \bar{\varepsilon}\right]}{\mu} \quad \text{and} \quad \lim_{d\to\infty} L_0^* = \frac{e_0 + \mathbb{E}\left[\tilde{\varepsilon}|\tilde{\varepsilon} \leq \bar{\varepsilon}\right]}{\mu}.$$

Example 3 If the macroeconomic random shock $\tilde{\varepsilon}$ is uniformly distributed on $[-\delta, \delta]$ for some $\delta > 0$, then in the absence of innovation the optimal carbon emissions cap is

$$E_0^* = \frac{e_0}{1 + \mu d},$$

while the optimal price controls are

$$U_0^* = d\left(\frac{e_0}{1+\mu d} + \frac{\delta}{2+\mu d}\right) \quad \text{and} \quad L_0^* = d\left(\frac{e_0}{1+\mu d} - \frac{\delta}{2+\mu d}\right).$$

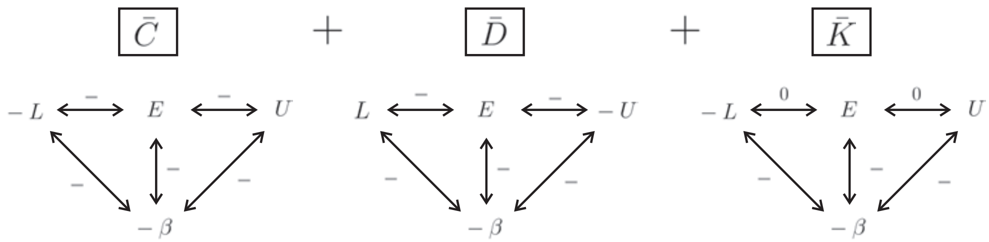

Figure 5: Complementarity Relationships in Social Cost $\overline{SC} = -\bar{W}$.

Using the same values for μ, σ_θ, c, d, e_0, and λ as in Example 1 and Example 2, in the absence of innovation we obtain an optimal emissions cap $E_0^* \approx 12.2 \cdot 10^9$ tCO_2 (as in Example 2) with optimal price controls $(L_0^*, U_0^*) \approx (38, 42)$ \$/tCO_2. At an innovation effectiveness of $\beta \approx 0.167 \cdot 10^{-3}\,(tCO_2)^2/\$^2$, and with $\delta = (10\%) \cdot e_0$ as in Example 2, the optimal emissions cap becomes $E^*(\beta) \approx 11.5 \cdot 10^9$ tCO_2 with the loosened price controls $(L^*(\beta), U^*(\beta)) \approx (42.8, 48.6)$ \$/tCO_2, for $\lambda = 0$ (resp. $E^*(\beta) \approx 11.9 \cdot 10^9$ tCO_2 and $(L^*(\beta), U^*(\beta)) \approx (37.2, 41.3)$ \$/tCO_2 for $\lambda = 1$). □

Comparative Statics Analysis

The proof of the last parts of Proposition 4 sheds further light on how the optimal regulatory scheme adjusts as some of its components are adjusted. In other words, the questions we would like to answer now are of the sort, 'what happens to the optimal price floor and the optimal emissions cap when the price ceiling is changed?' The latter adjustment may be needed for political reasons or for harmonizing between different cap-and-trade schemes in neighboring countries, despite the *prima facie* welfare losses in a single country.

As pointed out by Milgrom and Roberts (1990), based on earlier findings by Topkis (1968), among others, the monotonicity of optimal decisions (on lattices) critically depends on the supermodularity properties of the objective function in the decision variables and parameters.[16] Variable and/or parameter transformations may be used in case supermodularity does not obtain under the initial problem parametrization (Strulovici and Weber 2008). The latter turn out to be very simple in our context, as several simple sign reversals are enough to establish supermodularity of each of the components ($-\bar{C}$, $-\bar{D}$, and $-\bar{K}$) of the expected social welfare $\bar{W}$.

[16]Milgrom and Shannon (1994) show that quasi-supermodularity of the objective function is a sufficient (and in some sense necessary) condition for the monotonicity of solutions in parameters.

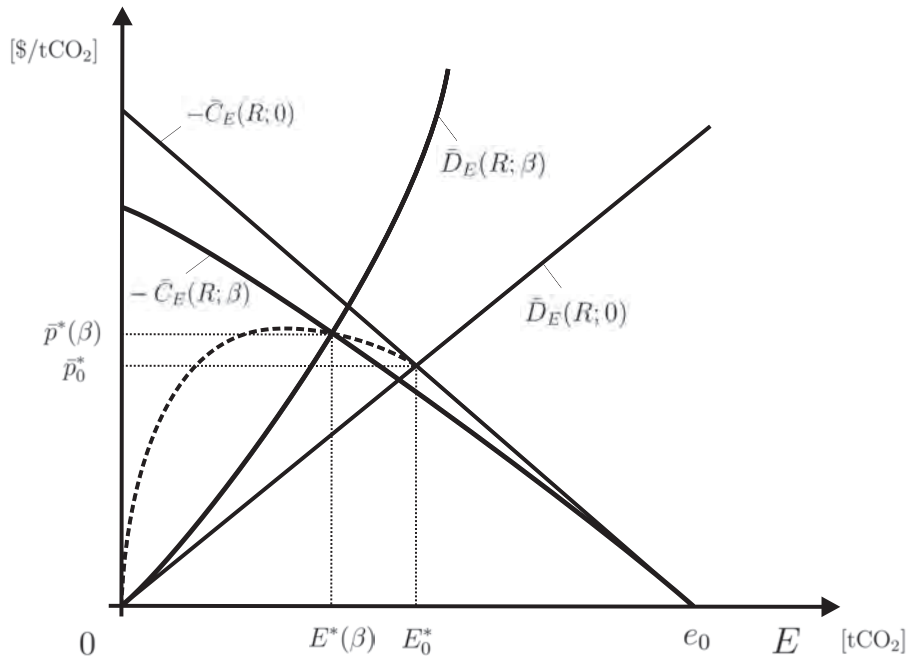

Figure 6: Expected Market Price $\bar{p}^*(\beta)$ as a Function of Innovation Effectiveness β.

The complementarity relationships between the decision variables and innovation effectiveness as well as marginal environmental damage, as determined by the sign of the cross-partial derivatives, are summarized by the diagrams in Figure 5. We see that both $-\bar{C}$ and $-\bar{K}$ are supermodular (have positive cross-partial derivatives) in $(E, -L, U, -\beta)$, whereas $-\bar{D}$ is supermodular in $(E, L, -U, D)$. Thus, when expected damages dominate in the social welfare, i.e., when d is large, then the monotone comparative statics obtain according to the complementarity properties of $\bar{D}$. When d is small, then the complementarity properties of $\bar{C}$ determine the comparative statics. Figure 5 is also useful for determining the direction of adjustments to the remaining policy instruments when one of them is changed exogenously. For example, when the price ceiling U is decreased and the slope of marginal environmental damage cost d is fairly large, then the optimal emissions cap decreases (same direction as change in price cap, for the relation between E and U in $-\bar{D}$ has a positive sign, as indicated by the '+' at the corresponding arrow) and the optimal price floor increases (opposite direction compared to change in price cap, as the relation between E and U in $-\bar{D}$ has a negative sign).

4 Policy Implications

A substantial private investment is needed to significantly reduce carbon emissions into the atmosphere. Effective regulatory schemes therefore need to take into account not only the firms' emissions decisions, but also the return on their R&D investments. A price floor guarantees a minimum return on innovation, whereas a price cap reduces the volatility in the aggregate abatement cost. The last section has shown that changes in the firms' propensity to innovate are likely to have a profound impact on the optimal design of carbon taxes as well as cap-and-trade markets, with or without price controls. It has also made clear that a higher innovation effectiveness tends to increase the attractiveness of cap-and-trade schemes vs. carbon taxes.[17] This is due to the fact that when prices increase, the substitution of emissions uncertainty for price uncertainty by imposing a tighter quantity control serves as an additional innovation incentive, while when prices decrease, the savings in abatement cost become so large that the regulator's only worry is the uncertainty in environmental damages, thus calling for quantity control.

The introduction of price controls in cap-and-trade markets, while superior to basic cap-and-trade from a purely mathematical point of view, is subject to a number of political considerations. First, the determination of a price cap in a political process is likely to lead to substantial influence activities by affected parties during the course of the legislative process, which may therefore produce price caps that are too low or price floors that are too high in the form of 'political compromises.' Second, environmental damages depend on aggregate carbon and other greenhouse gas emissions irrespective of their origin. This implies the need for a coordinated response and therefore government intervention, and international cooperation. A negative side-effect of price controls may be that they create challenges for the harmonization of cap-and-trade schemes with different price controls, and may even lead to arbitrage opportunities in cross-border trade of emissions permits.

Given the above caveats, what are the potential benefits of additional price controls?[18]

[17]Both taxes and cap-and-trade markets implement a price for carbon emissions, encouraging firms to switch to low-carbon technologies and to develop better carbon-abatement technologies. They also both raise funds (directly in the case of a carbon tax, indirectly using an emissions permit system) which can be used to mitigate environmental damages or to help other countries achieve common emissions-control targets.

[18]Indirect methods of 'price stabilization,' for example through the use of buffer stocks, were suggested in great detail by Newbery and Stiglitz (1981). In this spirit, it may be possible to relax direct price controls if one allows for emissions banking to create buffers moderating price fluctuations that would

Price caps reduce expected aggregate costs as well as the social cost of innovation, as can be seen directly from Eqs. (18) and (20). At the same time they tend to increase the expected environmental damages. Decreasing an existing price cap decreases the optimal emissions cap, as long as d is small. For large d, the opposite holds true. Thus, the degree to which a more ambitious policy can be pursued by introducing a price cap depends on the relative magnitude of the environmental damage cost. Price floors, on the other hand, offer a government-backed minimum value for emissions permits. This encourages innovation. Price floors also tend to reduce carbon-price volatility, thus increasing the emissions volatility and therefore the expected environmental damage, all else being equal. However, because of the higher innovation, it is to be expected that firms abate more carbon than before, compensating for the increased in the environmental damages due to the emissions-volatility increase.

5 Conclusion

In the absence of innovation, the classical Weitzman (1974) result states that under uncertainty the relative magnitude of marginal abatement costs and marginal environmental damage costs is crucial for deciding between tax-based or quantity-based policy instruments. When marginal environmental damage cost d is larger than the (expected) marginal abatement costs $1/\mu$, quantity-based regulation is preferable. The best instrument aims to parallel the marginal welfare as a function of the uncertainty. Weitzman's classical framework allows only for one degree of freedom, either the choice of the emissions cap or the choice of the carbon price. Introducing additional degrees of freedom through price controls in a cap-and-trade market allows one first to replicate each of the two simple schemes and then to improve welfare over both schemes.

We assume that firms can invest in innovation, and thus reduce the cost of mitigation efforts. This enables them to mitigate more carbon at the same price of carbon. With additional mitigation opportunities the marginal abatement cost is reduced. This shifts the tradeoff between marginal abatement cost and marginal environmental damage cost. The optimal emissions cap decreases in the innovation effectiveness. In the presence of innovation we observe two additional results. First, carbon prices create incentives for in-

otherwise result from the shocks in BAU emissions levels driven by macroeconomic uncertainty. A multi-period emissions-trading framework is needed to address this question.

novation in mitigation technologies. A welfare-optimal carbon policy targets an emissions level at which the innovation-enhanced marginal mitigation cost curve (considering expected innovation) intersects the damage cost curve that includes the additional benefits from incentives for innovation. As a result, the carbon price in a world with innovation can be higher than in a world without innovation. Second, the model shows that with increasing innovation, price controls are tightened when marginal environmental damage costs are low, and relaxed when these costs are large. Innovation creates mitigation opportunities that reduce the slope of the mitigation cost curve and therefore make the optimal instrument look more like a cap (e.g. wider spreads).

In the current discussion on price caps and floors, the analysis focuses often on static models. Including additional dimensions can materially alter the results. For example, the potential for innovation can increase the level at which caps and floors are set. We note that the analysis in this paper neglected several global effects of price caps, such as the question of what happens when they are set in a world of uncertain fuel and technology prices. The political debate surrounding price controls as additional policy instruments is complex. For example, Pizer (2002), among others, pointed out that price caps can increase the likelihood of governments accepting more stringent targets. On the other hand, price caps, through implicit borrowing from future periods, may reduce the incentive for governments or private companies to comply with emissions targets and can subsequently increase incentives to deviate from longer-term emissions targets so as to reduce the cost of debt. Additional models are required to examine such effects in greater detail.

Acknowledgments

The authors would like to thank Michael Grubb, David Newbery, an anonymous reviewer, as well as participants of the Energy and Environment Seminar at the University of Cambridge for helpful comments and discussion. Support by the British Academy, the Precourt Energy Efficiency Center, and the UK Research Councils is gratefully acknowledged. A major portion of this research was conducted while the first author was an official visitor at the Department of Economics, University of Cambridge.

Appendix: Proofs

Proof of Proposition 1. (i) Let $R = (e_0, \tau, \tau)$. The corresponding expected social welfare,

$$\bar{W}(\tau;\beta) = -\frac{\mu}{2}\left(1+\beta\tau^2\right)\tau^2 - \frac{d}{2}\left[\sigma_\varepsilon^2 + \left(e_0 - \mu\tau\left(1+\beta\tau^2\right)\right)^2\right] - \lambda\,\frac{\mu\beta\tau^4}{4},$$

is strictly concave in the tax level τ. In the absence of innovation it is $\beta = 0$, so that

$$\bar{W}(\tau;0) = -\frac{\mu\tau^2}{2} - \frac{d}{2}\left[\sigma_\varepsilon^2 + (e_0 - \mu\tau)^2\right],$$

and the unique optimal tax becomes $\tau_0^* = de_0/(1+\mu d)$. (ii) For $\beta > 0$, the optimal tax $\tau^*(\beta)$ is determined by the first-order necessary optimality condition $\bar{W}_\tau(\tau^*(\beta);\beta) = 0$.[19] Differentiating this condition implicitly with respect to β yields

$$\frac{d\tau^*(\beta)}{d\beta} = -\frac{\bar{W}_{\tau\beta}(\tau^*(\beta);\beta)}{\bar{W}_{\tau\tau}(\tau^*(\beta);\beta)}.$$

Since $\bar{W}_{\tau\tau}(\tau^*(\beta);\beta) < 0$ at the welfare-maximizing tax level, the optimal tax $\tau^*(\beta)$ is decreasing if and only if $\bar{W}_{\tau\beta}(\tau^*(\beta);\beta) < 0$. Combining

$$\bar{W}_{\tau\beta}(\tau;\beta) = \mu\tau^3\left[\frac{3de_0}{\tau} - 2\left(1+\mu d\left(2+3\beta\tau^2\right)\right) + \lambda\right]$$

with the fact that by the first-order condition

$$\frac{de_0}{\tau} = \frac{1+(2+\lambda)\beta\tau^2 + \mu d(1+\beta\tau^2)(1+3\beta\tau^2)}{1+3\beta\tau^2},$$

we obtain that $\bar{W}_{\tau\beta} < 0$ if and only if

$$\mu d > \frac{1-\lambda}{\left(1+3\beta\tau^2\right)^2}.$$

(iii) Since $\lim_{\beta\to\infty}\bar{W}_\tau(\tau;\beta)/\beta^2 = -3\mu^2 d\tau^5 = 0$ at the optimal tax $\tau = \tau^*(\infty)$, we must have that $\tau^*(\infty) = 0$, i.e., the optimal tax converges to zero as perfect abatement becomes free, which concludes our proof. ∎

Proof of Proposition 2. (i) Let $R = (E, 0, \infty)$. The expected social welfare in this case is

$$\bar{W}(E;\beta) = -\mathbb{E}\left[\frac{\mu}{2}\left(1+\beta p^2(\tilde{\varepsilon},E)\right)p^2(\tilde{\varepsilon},E) + \frac{dE^2}{2} + \frac{\lambda\mu\beta p^4(\tilde{\varepsilon},E)}{4}\right],$$

[19] By Abel's well-known impossibility theorem (see, e.g., Hungerford 1974, p. 308) a closed-form solution for $\tau^*(\beta)$ cannot be expected.

where $p(\varepsilon, E) = p_0(\varepsilon, E) - \Delta(p_0(\varepsilon, E), \beta)$ as in Eq. (12). In the absence of innovation, i.e., for $\beta = 0$, it is

$$\bar{W}(E; 0) = -\frac{(e_0 - E)^2}{2\mu} - \frac{dE^2}{2} - \frac{\sigma_\varepsilon^2}{2\mu},$$

strictly concave in E, leading to a unique optimal emissions cap of $E_0^* = e_0/(1 + \mu d)$. (ii) For $\beta > 0$, the optimal emissions cap $E^*(\beta)$ is implicitly determined by the first-order necessary optimality condition

$$\bar{W}_E(E^*(\beta), \beta) = 0. \tag{23}$$

Differentiating Eq. (23) on both sides with respect to β, we obtain that

$$\frac{dE^*(\beta)}{d\beta} = -\frac{\bar{W}_{E\beta}(E^*(\beta); \beta)}{\bar{W}_{EE}(E^*(\beta); \beta)}.$$

By differentiating the market-clearing condition (21) it is

$$p_E = -\frac{1}{\mu}\frac{1}{1 + 3\beta p^2}, \tag{24}$$

$$p_\beta = -\frac{p^3}{1 + 3\beta p^2}, \tag{25}$$

and

$$p_{E\beta} = \frac{3p^2}{\mu}\frac{1 + \beta p^2}{(1 + 3\beta p^2)^3}, \tag{26}$$

so that

$$\begin{aligned} \bar{W}_{E\beta}(E^*(\beta); \beta) &= \mu\mathbb{E}\left[-(\lambda + 2\mu)\left(p^3 + 3\beta p^2 p_\beta\right) p_E - (\lambda + 2\mu)\beta p^3 p_{E\beta} - p_\beta p_E - p p_{E\beta}\right] \\ &= \mathbb{E}\left[\frac{\left(-4 - 3\beta p^2 + (2\mu + \lambda)(1 - \beta^2 p^4)\right) p^3}{\mu(1 + 3\beta p^2)^3}\right] < 0 \end{aligned}$$

for all $\lambda \in [0, 1]$ and all $\mu > 0$ (with $p = p(\tilde{\varepsilon}, E^*(\beta))$). Since $\bar{W}_{EE}(E^*(\beta); \beta) < 0$ at the welfare-maximizing emissions cap, the fact that $\bar{W}_{E\beta} < 0$ implies that the optimal emissions cap $E^*(\beta)$ is decreasing. (iii) Note first that $p(\infty) = 0$, i.e., the market price for carbon vanishes for $\beta \to \infty$, which is obtained by taking the corresponding limit in the market-clearing condition (21). Thus, taking the limit in the first-order condition (23) implies that $E^*(\infty) = 0$, which concludes our proof. ∎

Proof of Proposition 3. (i) Given any $\beta > 0$, let $\bar{W}_{\text{Tax}}(\tau; \beta)$ and $\bar{W}_{\text{Basic C\&T}}(E; \beta)$ be the expected welfare for a pure tax of τ and an emissions cap of E, respectively. At the optimal levels $\tau^*(\beta)$ and $E^*(\beta)$, an application of the envelope theorem yields that

$$\frac{d}{d\beta}\left(\bar{W}^*_{\text{Tax}}(\beta) - \bar{W}^*_{\text{Basic C\&T}}(\beta)\right) = \frac{\mu}{2}\left(1-\frac{\lambda}{2}\right)\left(\left.\frac{\left(1+\left(\frac{2+\lambda}{2-\lambda}\right)\beta\tau^2\right)\tau^4}{1+3\beta\tau^2}\right|_{\tau=\tau^*(\beta)} - \mathbb{E}\left[\left.\frac{\left(1+\left(\frac{2+\lambda}{2-\lambda}\right)\beta\tilde{p}^2\right)\tilde{p}^4}{1+3\beta\tilde{p}^2}\right|\tilde{p}=p(\tilde{\varepsilon},E^*(\beta))\right]\right),$$

where $\bar{W}^*_{\text{Tax}}(\beta) = \bar{W}_{\text{Tax}}(\tau^*(\beta);\beta)$, $\bar{W}^*_{\text{Basic C\&T}}(\beta) = \bar{W}_{\text{Basic C\&T}}(E^*(\beta);\beta)$, and $p(\tilde{\varepsilon},E^*(\beta)) = p_0(\tilde{\varepsilon},E^*(\beta)) - \Delta(p_0(\tilde{\varepsilon},E^*(\beta)),\beta)$ as in Eq. (12). Taking the limit for $\beta\to 0+$ on both sides of the last equation, we obtain

$$\left.\frac{d}{d\beta}\right|_{\beta=0}\left(\bar{W}^*_{\text{Tax}}(\beta) - \bar{W}^*_{\text{Basic C\&T}}(\beta)\right) = \frac{\mu}{2}\left(1-\frac{\lambda}{2}\right)\left((\tau_0^*)^4 - \mathbb{E}\left[(p_0(\tilde{\varepsilon},E_0^*))^4\right]\right) < 0,$$

since $\tau_0^* = \mathbb{E}\left[p_0(\tilde{\varepsilon},E_0^*)\right]$ by Eq. (22), and $\mathbb{E}\left[p_0(\tilde{\varepsilon},E_0^*)\right] < \mathbb{E}\left[(p_0(\tilde{\varepsilon},E_0^*))^4\right]$ by Jensen's inequality (as long as there exists nontrivial macroeconomic uncertainty $\tilde{\varepsilon}$, so that $\sigma_\varepsilon > 0$). From the continuity of the derivative of $\bar{W}^*_{\text{Tax}}(\beta) - \bar{W}^*_{\text{Basic C\&T}}(\beta)$ we can therefore conclude that there exists a $\beta_0 > 0$, such that

$$\frac{d}{d\beta}\left(\bar{W}^*_{\text{Tax}}(\beta) - \bar{W}^*_{\text{Basic C\&T}}(\beta)\right) < 0$$

for all $\beta\in(0,\beta_0)$. (ii) By taking the limit for $\beta\to\infty$ in Eq. (16) we obtain that $\Delta(p_0,\infty) = p_0$ for any p_0. Hence, Eq. (12) implies that

$$\lim_{\beta\to\infty} p(\varepsilon,E^*(\beta)) = \lim_{\beta\to\infty}\left(p_0(\varepsilon,E^*(\beta)) - \Delta(p_0(\varepsilon,E^*(\beta)),\beta)\right) = 0$$

for all ε. Thus, using part (iii) of Proposition 2 together with Eq. (13) and the market-clearing condition (21), it is

$$\lim_{\beta\to\infty} p_0(\varepsilon,E^*(\beta)) = \mu\lim_{\beta\to\infty}\beta\left(p(\varepsilon,E^*(\beta))\right)^3 = e_0+\varepsilon$$

for all ε. Therefore,

$$\lim_{\beta\to\infty}\beta(p(\varepsilon,E^*(\beta)))^4 = \left(\lim_{\beta\to\infty}\beta(p(\varepsilon,E^*(\beta)))^3\right)\left(\lim_{\beta\to\infty}p(\varepsilon,E^*(\beta))\right) = 0,$$

which implies (after a legitimate switch of limit and integration) in Eqs. (18)–(20) (for $R = (E^*(\beta),0,\infty)$) that

$$\lim_{\beta\to\infty}\bar{W}^*_{\text{Basic C\&T}}(\beta) = 0.$$

On the other hand, by part (iii) of Proposition 1 it is $\tau^*(\infty) = 0$, so that, using Eqs. (18) and (20), the social cost of innovation and aggregate abatement cost are zero. The key difference of pure taxation is that as innovation effectiveness goes to infinity, the (deterministic!) optimal tax level approaches zero, and at the same time the aggregate abatement approaches e_0, so that, using Eqs. (6) and (17), the aggregate emissions output becomes

$$\lim_{\beta\to\infty} Q(\tau^*(\beta),\varepsilon) = \lim_{\beta\to\infty} \left(e_0+\varepsilon-\mu\tau^*(\beta)\left(1+\beta(\tau^*(\beta))^2\right)\right) = \varepsilon$$

for all ε. Expected aggregate environmental damages are therefore equal to $-d\sigma_\varepsilon^2/2$, so that

$$\lim_{\beta\to\infty} \bar{W}^*_{\text{Tax}}(\beta) = -\frac{d\sigma_\varepsilon^2}{2} < 0 = \lim_{\beta\to\infty} \bar{W}^*_{\text{Basic C\&T}}(\beta),$$

which concludes our proof. ■

Proof of Proposition 4. (i) we first consider the situation without innovation, where $\beta = 0$ and where $\bar{W}(R;0)$ is the expected social welfare without innovation. Using the Leibniz rule, we obtain the first-order necessary optimality condition

$$\frac{\partial \bar{W}(R;0)}{\partial E} = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \left(\frac{1}{\mu}(e_0+\varepsilon-E)-dE\right) dG(\varepsilon) = 0,$$

which is equivalent to

$$E_0^* = \frac{e_0+\mathbb{E}\left[\tilde{\varepsilon}|\underline{\varepsilon}\le\tilde{\varepsilon}\le\bar{\varepsilon}\right]}{1+\mu d}.$$

Similarly, we obtain

$$\frac{\partial \bar{W}(R;0)}{\partial U} = \int_{\bar{\varepsilon}}^{\infty} \left(-\mu U+\mu d(e_0+\varepsilon-\mu U)\right) dG(\varepsilon) = 0,$$

which is equivalent to

$$U_0^* = d\cdot\frac{e_0+\mathbb{E}\left[\tilde{\varepsilon}|\tilde{\varepsilon}\ge\bar{\varepsilon}\right]}{1+\mu d},$$

and, analogously,

$$L_0^* = d\cdot\frac{e_0+\mathbb{E}\left[\tilde{\varepsilon}|\tilde{\varepsilon}\le\underline{\varepsilon}\right]}{1+\mu d}.$$

(ii),(iii) We examine the supermodularily properties of the expected welfare $\bar{W} = -\bar{C} - \bar{D} - \bar{K}$ for each of its components. Consider first

$$\bar{C}(R;\beta) = \frac{\mu}{2}\int_{-\infty}^{\infty} \left(1+\beta p^2(\varepsilon,R)\right) p^2(\varepsilon,R)\, dG(\varepsilon),$$

with

$$\bar{C}_\beta(R;\beta) = \frac{\mu}{2}\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \left(2(1+2\beta p^2)pp_\beta + p^4\right) dG(\varepsilon),$$

where p_β is given in Eq. (25). Using Eqs. (24)–(26), we therefore find that

$$\bar{C}_{\beta L} = -\frac{\mu^2(1+4\beta L^2)L^7 G'(\underline{\varepsilon})}{1+3\beta L^2} < 0 < \frac{\mu^2(1+4\beta U^2)U^7 G'(\bar{\varepsilon})}{1+3\beta U^2} = \bar{C}_{\beta U},$$

and

$$\bar{C}_{\beta E} = 2\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \frac{(1+3\beta p^2+3\beta^2 p^4)p^3}{(1+3\beta p^2)^3} dG(\varepsilon) + \frac{\mu U^4(1+4\beta U^2)G'(\bar{\varepsilon})}{1+3\beta U^2} - \frac{\mu L^4(1+4\beta L^2)G'(\underline{\varepsilon})}{1+3\beta L^2}.$$

Since, by hypothesis, the macroeconomic uncertainty is nondecreasing on its support, it is $G'(\underline{\varepsilon}) \le G'(\bar{\varepsilon})$. Furthermore, it is easy to show that the function $\mu x^4(1+4\beta x^2)/(1+3\beta x^2)$ is strictly increasing in $x > 0$, so that indeed $\bar{C}_{\beta E} > 0$. In addition, $\bar{C}_{LU} = 0$, $\bar{C}_{EU} = -\mu U(1+2\beta U^2)G'(\bar{\varepsilon}) < 0 < \mu L(1+2\beta L^2)G'(\underline{\varepsilon}) = \bar{C}_{EL}$, which implies that $\bar{C}$ is submodular in $(E, -L, U, -\beta)$. Consider now

$$\bar{K}(R;\beta) = \frac{\lambda\mu\beta}{4}\int_{-\infty}^{\infty} p^4 dG(\varepsilon),$$

so that

$$\bar{K}_\beta(R;\beta) = \frac{\lambda\mu}{4}\int_{-\infty}^{\infty} p^4 dG(\varepsilon) + \lambda\mu\beta\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} p^3 p_\beta dG(\varepsilon).$$

Using Eqs. (24)–(26), it is therefore

$$\bar{K}_{\beta U} = -\frac{\lambda\mu^2\beta U^9}{1+3\beta U^2} < 0 < \frac{\lambda\mu^2\beta L^9}{1+3\beta L^2} = \bar{K}_{\beta L}$$

and

$$\bar{K}_{\beta E} = -\lambda\left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \frac{(1-3\beta^2 p^4)p^3}{(1+3\beta p^2)^3} dG(\varepsilon) + \frac{L^3 G(\underline{\varepsilon})}{1+3\beta L^2} + \frac{U^3(1-G(\bar{\varepsilon}))}{1+3\beta U^2}\right) < 0.$$

In addition, $\bar{K}_{LU} = \bar{K}_{EL} = \bar{K}_{EU} = 0$, so that we have shown that $\bar{K}$ is submodular in $(E, -L, U, -\beta)$. Lastly, consider

$$\begin{aligned}\bar{D}(R;\beta) &= \frac{d}{2}\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} E^2 dG(\varepsilon) + \frac{d}{2}\int_{-\infty}^{\underline{\varepsilon}} \left(e_0+\varepsilon-\mu L\left(1+\beta L^2\right)\right)^2 dG(\varepsilon) \\ &\quad +\frac{d}{2}\int_{\bar{\varepsilon}}^{\infty} \left(e_0+\varepsilon-\mu U\left(1+\beta U^2\right)\right)^2 dG(\varepsilon),\end{aligned}$$

so that

$$\begin{aligned}\bar{D}_\beta(R;\beta) &= -\mu d\left[L^3\int_{-\infty}^{\underline{\varepsilon}} \left(e_0+\varepsilon-\mu L(1+\beta L^2)\right) dG(\varepsilon)\right. \\ &\qquad \left. +U^3\int_{\bar{\varepsilon}}^{\infty} \left(e_0+\varepsilon-\mu U(1+\beta U^2)\right) dG(\varepsilon)\right] \\ &= -\mu d\left[L^3\int_{-\infty}^{\underline{\varepsilon}} (E+\varepsilon-\underline{\varepsilon})\, dG(\varepsilon) + U^3\int_{\bar{\varepsilon}}^{\infty} (E+\varepsilon-\bar{\varepsilon})\, dG(\varepsilon)\right].\end{aligned}$$

Thus, we find

$$\begin{aligned}
\bar{D}_{\beta L} &= \mu d(1 - G(\underline{\varepsilon}))L^2 \left[3\left(E + \mathbb{E}\left[\tilde{\varepsilon}|\,\tilde{\varepsilon} \leq \underline{\varepsilon}\right] - \underline{\varepsilon}\right) - \mu L(1 + 3\beta L^2)(1 + EG'(\underline{\varepsilon}))\right] > 0, \\
\bar{D}_{\beta U} &= -\mu d(1 - G(\bar{\varepsilon}))U^2 \left[3\left(E + \mathbb{E}\left[\tilde{\varepsilon}|\,\tilde{\varepsilon} \geq \bar{\varepsilon}\right] - \bar{\varepsilon}\right) - \mu U(1 + 3\beta U^2)(1 + EG'(\bar{\varepsilon}))\right] < 0,
\end{aligned}$$

and

$$\bar{D}_{\beta E} = \mu d\left[U^3 G'(\bar{\varepsilon}) - L^3 G'(\underline{\varepsilon})\right] > 0.$$

Moreover, $\bar{D}_{LU} = 0$, and $\bar{D}_{EL} = -dEG'(\underline{\varepsilon}) < 0 < dEG'(\bar{\varepsilon}) = \bar{D}_{EU}$, which, together with the previous inequalities, implies that $\bar{D}$ is submodular in $(E, L, -U, -\beta)$. Hence, for small damages $\bar{W}$ is supermodular in $(E, -L, U, -\beta)$, and for large damages $\bar{W}$ is supermodular in $(E, L, -U, -\beta)$. This implies that $E^*(\beta)$ is decreasing. Second, for small environmental damages, $L^*(\beta)$ is increasing and $U^*(\beta)$ is decreasing (i.e., more stringent price control). Third, for high environmental damages, $L^*(\beta)$ is decreasing and $U^*(\beta)$ is increasing (i.e., less stringent price control). ■

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