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Allocating Transmission to Mitigate Market Power in Electricity Networks

Richard Gilbert, Karsten Neuhoff and David Newbery

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Allocating Transmission to Mitigate Market Power in Electricity Networks*

Richard Gilbert — University of California
Karsten Neuhoff — University of Cambridge
David Newbery — University of Cambridge

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Abstract

We ask under what conditions transmission contracts increase or mitigate market power. We show that the allocation process of transmission rights is crucial. In an efficiently arbitraged uniform price auction generators will only obtain contracts that mitigate their market power. However, if generators inherit transmission contracts or buy them in a 'pay-as-bid' auction, then these contracts can enhance market power. In the two-node network case banning generators from holding transmission contracts that do not correspond to delivery of their own energy mitigates market power. Meshed networks differ in important ways as constrained links no longer isolate prices in competitive markets from market manipulation. The paper suggests ways of minimising market power considerations when designing transmission contracts.

1 Introduction

Limited transmission capacity has been an impediment to creating competitive electricity markets in Europe and the United States. In many cases the interconnection capacity between regions or countries was developed to provide security rather than to facilitate energy trade. In liberalised markets where consumers are free to buy from out-of-area generators, this capacity is often inadequate and must be rationed or priced. Transmission constraints isolate electricity markets and limit the number of generators competing to supply local consumers. Increasing

*gilbert@econ.berkeley.edu, karsten.neuhoff@econ.cam.ac.uk, david.newbery@econ.cam.ac.uk. Research support from Cambridge-MIT Institute under the project 045/P *'Promoting Innovation and Productivity in Electricity Markets'*, Gottlieb-Daimler Foundation, and Economic and Social Research Council (award R42200034307) is gratefully acknowledged. Sidgwick Avenue, Cambridge CB3 9DE, England Phone 44 1223 335200, Fax 44 1223 335299

interconnector capacity has an additional advantage as it reduces local market power (Joskow and Schmalensee, 1983). In many European countries, the concentration in generation is considerable, and the dominant producer is required to produce for a large fraction of the year to make up any shortfall between demand and available supply from imports and other producers. As the short-run elasticity of electricity demand is extremely low, dominant producers have very substantial market power when they are the required residual supplier. The Californian electricity crisis of December 2000 provides stark evidence that market power can be a serious problem in tight markets even when they appear unconcentrated (Joskow and Kahn, 2002) .

Mitigating market power is therefore high on the agenda for regulators on both sides of the Atlantic. In the longer run, increased interconnection will reduce effective concentration, as should entry by new generating companies, and possibly regulatory pressure encouraging divestment, as in Britain. In the short run, though, interconnection capacity is limited and fixed. When the transmission constraints are binding then transmission capacity becomes valuable and revenues from financial transmission contracts or exercise of physical transmission rights influence generators' production decisions. The key policy question that this paper addresses is how such scarce interconnection capacity should be made available to minimise the damaging effects of market power of current levels of generation concentration. We are specifically concerned with the design of auctions and markets (both spot and contract) to mitigate market power.

In a nodal pricing system, or where interconnection connects neighboring areas facing different spot markets, transmission constraints expose market participants to locational price differences. Transmission contracts provide access to scarce transmission lines and provide financial hedges against price risk. In addition, prices provide information to guide the expansion of transmission capacity (Hogan, 1992). One immediate question confronting regulators is whether, and if so, under what conditions, transmission contracts increase or mitigate the market power of electricity generators.

The literature on the analysis of market power in transmission networks has developed in a series of papers addressing particular problems. These provide considerable insight, though the robustness of their findings and the implications for network management and market design are rather scattered and hard to assess. The paper therefore aims to present a systematic analysis of the best way to mitigate market power in constrained networks. This question provides a natural and important organising principle for investigating the effects of different arrangements on generator behaviour and consumer welfare.

Borenstein, Bushnell and Stoft (2000) show that it can be profitable for generators to withhold output in order to constrain a transmission line that would not have been constrained under perfect competition. Borenstein et al (1996) cite empirical evidence for Northern California to this effect. Oren (1997) presents an alternative scenario with the transmission constraint located between two strategic generators in a three-node network. Stoft (1998) solves the corresponding Cournot game and Joskow and Tirole (2000) give the interpretation: the transmission configuration can turn the outputs of generators at two different nodes into 'local complements', thereby increasing the incentive for a generator to withhold output, as this constrains the output of the other generator and increases price levels. Hogan (1997) shows if strategic generators own generation assets at node A and B of a three node network, then they might increase output at node A relative to a competitive scenario if loop flows reduce the total energy delivered and increase prices at node B. Harvey and Hogan (2000) compare market power under nodal and zonal congestion management and conclude that the impact of market power is always weakly lower under nodal pricing than if both nodes are aggregated into a single zone. Further effects of market power in networks are addressed by Neuhoff (2003) modelling the separation of markets for physical transmission capacity and energy spot markets for unconstrained, partially constrained and constrained links. He concludes that market power is mitigated if the separate markets are combined in a nodal design or market-splitting approach.

An additional dimension is added if generators own and/or bid for transmission contracts. To maintain simplicity this paper only addresses situations when constraints are binding irrespective of the existence of contracts. Stoft (1999) shows that transmission contracts may curb market power in a two-node network. In contrast Hogan (1997) presents an example of a strategic generator with assets at two nodes of a three-node network where transmission contracts enhance his market power. Joskow and Tirole (2000) show that a generator with assets at one node of a three-node network can increase output beyond the competitive output to increase rents from transmission contracts, with negative welfare effects.

Joskow and Tirole (2000), subsequently referred to as J&T, provide the most impressive and comprehensive treatment of the effect of transmission contracts, on which much of this paper is built. They provide an extensive analysis of the difference between physical and financial transmission contracts. Bushnell (1999) argues that it is important to ensure that scarce transmission capacity is not withheld, by imposing 'use-it-or-lose-it' conditions. This last point is widely accepted and practised by European regulators,[1] though J&T further argue that it may be difficult in practice to enforce such a policy. Assuming that 'use-it-or-lose-it' conditions are successfully imposed, J&T show that in a two-node network with the transmission link constrained, there is an equivalence between financial transmission contracts and physical transmission contracts. For meshed networks J&T discuss the difficulties of implementing physical transmission contracts and

[1]For example, the Dutch electricity regulator has taken steps against companies that did not use all of the transmission capacity allocated to them. See http://www.nma-dte.nl

Neuhoff (2003) shows that financial transmission contracts based on nodal pricing are preferable for reducing generators' market power. Financial transmission contracts are also preferable in temporarily unconstrained two-node and meshed networks. Trading arrangements required for physical transmission contracts typically create additional constraints which reduce net-demand elasticities generators face and thereby enhance their ability to exercise market power.

In their paper, J&T focus on the incentives that ownership of financial transmission contracts provide to generators in constrained simple and meshed networks. We build on these assumptions to provide a more comprehensive and robust range of cases, to assess the interaction between allocation and re-trading of contracts, and to examine the choice of auction and market design.

Joskow and Tirole start with the two-node case with a single monopolist at one node facing competitive generators at the other node, and give examples in which contracts either enhance or mitigate market power. In our interpretation transmission contracts serve the same function as forward contracts for energy. A transmission contract links the value of generation at one node to the price at another node. If the price at the other node is given (or hedged in local spot markets), then a transmission contract is a forward contract for energy.

If a dominant generator at an importing node imports energy with a transmission contract and sells it in the local spot market, this will increase the total volume of energy he sells at the spot market price and increases his incentive to withhold domestic output to increase spot prices. Market power is therefore enhanced if importing generators hold transmission contracts. In contrast, in a two-node network if an exporting generator holds transmission contracts he can effectively pre-commit that part of his output, in the same way that selling in a forward market would pre-commit output. As with other forms of contracting generator output, this is pro-competitive as it reduces the fraction sold at the spot market price and hence the incentive to influence the spot price. Transmission contracts held by exporting generators mitigate market power. In two-node markets like Scotland-England or Germany-Netherlands the market power can therefore be mitigated by restricting generators from buying transmission contracts that do not correspond to delivery of energy from their power plants.

Joskow and Tirole also address the three-node network but conclude that "The difference between the two-and three-node networks is, we feel, more quantitative than qualitative; ..." (Joskow and Tirole, 2000, p. 479). However, loop flows in a meshed network make prices at different nodes interdependent even if transmission constraints are binding. If a generator holds a transmission contract to a node with a price that increases more due to withholding than the price at the production node, then the incentive to withhold output is increased. Joskow and Tirole showed that in a meshed network transmission contracts held by exporting generators can enhance market power. We think that is a crucial difference to the two-node case where the same contracts always mitigate market power. Our main result for meshed networks (which includes the two-node case) is that a perfectly arbitraged single-price auction ensures that contracts never enhance and may mitigate market power. Traders value transmission contracts more than generators because in their possession the contracts do not create additional production inefficiencies in the energy market. As a result traders will secure all the contracts.

Joskow and Tirole analyse a pay-as-bid auction by the system operator for the monopoly case. They show that generators play a mixed strategy in the pay-as-bid auction. We solve this case explicitly and extend it to the more commonly encountered oligopoly case, which differs in some important respects. Traders know only the expected distribution of generators' bids. This lack of information implies that traders can only arbitrage the price of transmission contracts in expectation and therefore bid less aggressively than in a uniform price auction. Generators then obtain variable amounts of transmission contracts that increase their market power.

Uniform price auctions seem preferable to pay-as-bid auctions as they allow traders to arbitrage away the extra market power that transmission contracts offer generators. However, one should be cautious before accepting that uniform price auctions suffice to address market power problems. The result only applies to the full information case and it remains an open question whether it would still apply with asymmetric information or uncertainty. The result also depends on the assumption that the energy spot market equilibrium price is predictable, as it is with Cournot competition and information about cost and demand characteristics (of the kind normally available to informed participants in electricity markets). If the energy market is modelled as a supply function equilibrium as in Green and Newbery (1992), then the Nash equilibrium in bidding strategies is no longer unique. As a result the value of transmission contracts to traders is uncertain. In that case the outcome may share some of the unfavorable outcomes of the pay-as-bid auction: traders bid less aggressively, allowing generators to obtain market power-enhancing contracts.

The working assumption of Cournot competition in that sense is most favorable for a regulatory minimalist approach to market intervention. Competition in supply functions might create uncertainty and information asymmetries, limiting the effectiveness of uniform price auctions. Whilst careful auction design can reveal private information and therefore reduce these information asymmetries, it may be more direct to use a simple auction design and explicitly ban generators from obtaining certain types of transmission contracts.

Joskow and Tirole discuss surveillance of 'gambling' behavior of generators to mitigate the risk that contracts may enhance market power. They conclude that 'regulatory surveillance of transmission-rights ownership that turns on 'gambling' or 'underhedging' behavior is likely to be difficult to implement under many real-world supply situations.' (J&T p. 469) Our analysis suggests that one possible solution could be to identify the reference network node whose price is least influenced by any generator's output decision, and to define all transmission contracts towards that node. Generators would be restricted to buying transmission contracts towards this reference node. If such a node can be identified (and to be useful its identity would need to be relatively stable over defined time periods), then all contracts should mitigate market power and our aim would have been achieved. Consumers can buy transmission contracts from the reference node to their off-take node such that the entire transmission risk is eliminated.

The paper also examines another related and practically important issue of trading in contracts.[2] Industry restructuring resulted in various legacy interconnection contracts held by generators and likewise discriminatory auctions allow generators to obtain contracts that enhance market power. An important question is whether subsequent trade in these contracts would resolve that problem. A monopolist would not sell contracts that enhance his market power. Whether oligopolists sell contracts depends both on the trading structure and on the initial allocation of contracts. If a well-defined final trading period exists then oligopolists will always sell some of their market power-enhancing contracts. If their initial contract holding is symmetric and sufficient trading periods exist, they will sell all these contracts. If retrading is possible after a discriminatory auction, then generators will sell some, but not all, of the contracts they obtained in the auction. However, this will not necessarily improve the final outcome, because it can induce generators to bid more frequently for transmission rights. We therefore conclude that contract trading does not in general resolve the problem of an inappropriate initial allocation of transmission contracts. Regulators should therefore consider the market power implications when grandfathering and auctioning contracts.

2 Energy market with importer market power

We assume that perfectly competitive generators with constant marginal costs c_1 are net exporters of electricity from node one. At node two, a total of n generators compete strategically and face convex demand. A transmission link with capacity K connects the nodes. (Figure 1).

At stage one of the game, transmission contracts are allocated either according to some rule (e.g. as legacy contracts) or in an auction. At stage two the market design could allow generators to re-trade transmission contracts. At stage three generators determine their outputs in the energy spot market. This replicates the current market structure in European countries where transmission auctions close before the spot market closes.

[2] Joskow and Tirole assess the case of one trader holding all contracts. If these contracts can enhance the market power of a monopolist generator and bilateral trade is possible, then the trader will sell all contracts to the generator. (J&T p. 459)

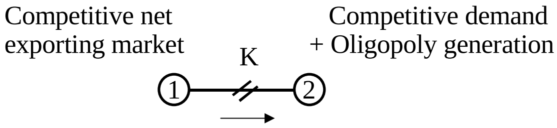

Figure 1: Oligopoly importing case

We solve the model backwards, starting in this section with the energy market. We assume that the transmission link is always used at full capacity so output decisions at node two do not influence prices at node one, which stay constant at marginal cost c_1. The price at node two depends on total supply, made up of imports K and local production $Q = \sum_i q_i$:

$$p_2 = P_2(Q + K), \qquad p_{2,comp} > c_1. \tag{1}$$

The generators choose output to maximise the profits, π_i, they obtain from selling energy in the local spot market, plus the revenue from their transmission contracts k_i:

$$\pi_i^{spot}(q_i, k_i) = P_2 q_i - C_i(q_i) + k_i\left(P_2 - c_1\right), \tag{2}$$

where $C_i(q)$ is total cost. The first order condition for the choice of q_i is

$$p_2 - C_i'(q_i) + (q_i + k_i)\frac{dP_2}{dQ}\frac{dQ}{dq_i} = 0, \tag{3}$$

(where dQ/dq_i captures a range of imperfectly competitive responses). From this

$$q_i + k_i = \frac{p_2 - C_i'}{-\frac{dP_2}{dQ}\cdot\frac{dQ}{dq_i}}. \tag{4}$$

Transmission contracts are also held by competitive traders or consumers. Compare two allocations, the second of which is identical to the first except that generator i has an additional Δ transmission contracts, and traders or consumers have Δ fewer in total. If k_i increases to $k_i + \Delta$, we wish to argue that then q_i necessarily decreases and so does Q. It is tempting to argue that the second step follows from a stability condition, for if by increasing q_i, Q were to fall then the price would rise, making further increases in q_i even more profitable. That line of argument is in general invalid, as it ignores the time sequence of decisions. The change in k_i happens before agents choose their output decisions, and they condition on that as well as the presumed output choices of other generators. To make further progress, we can either assume that output choices are Nash-Cournot (NC, $dQ/dq_i = 1$), or that $\frac{dQ}{dq_i}$ is constant (as would arise with a linear supply function model with linear marginal costs and linear demand).

Consider first the NC case, with $dQ/dq_i = 1$ in (3). During the reallocation of transmission contracts, (3) has to be satisfied, and can therefore be differentiated with respect to k_j:

$$\left((q_i + k_i)P_2'' + P_2'\right)\frac{dQ}{dk_j} + \delta_{ij}P_2' + \left(P_2' - C_i''(q_i)\right)\frac{dq_i}{dk_j} = 0, \tag{5}$$

where δ_{ij} takes the value 1 if $i = j$ and 0 otherwise. This can be rearranged to give

$$\frac{dq_i}{dk_j} = \frac{\delta_{ij} P_2' + [(q_i + k_i)P_2'' + P_2'] \frac{dQ}{dk_j}}{-[P_2' - C_i''(q_i)]}. \tag{6}$$

The denominator of (6) is positive as P_2' is negative, and efficient plant dispatch implies that $C_i''(q_i) \geq 0$. If $P_2''(q_i + k_i) + P_2'$ is negative, then it must follow that $dQ/dk_j < 0$. For suppose not, then $dq_i/dk_j < 0$, for all i, from (6), but $\frac{dQ}{dk_j} = \sum \frac{dq_i}{dk_j}$, giving a contradiction. Clearly $P_2''(q_i + k_i) + P_2'$ is negative for concave or linear demand, for then $P_2'' \leq 0$, but the second order condition from (6) by itself is not sufficient to show that it is negative more generally. Nevertheless, the conditions for $dQ/dk_j < 0$ are quite weak.

First consider the case of constant marginal costs: $C_i''(q_i) = 0$. Sum (6) over i after dividing by $-P_2'$:

$$\frac{dQ}{dk_j} = -1 - \left(n - (Q + \sum k_i) \frac{P_2''}{-P_2'}\right) \frac{dQ}{dk_j}. \tag{7}$$

It follows that $dQ/dk_j < 0$ if

$$\frac{QP_2''}{-P_2'} < (n+1) \frac{Q}{Q + \sum k_i}, \tag{8}$$

which is a weak condition on the degree of convexity of the demand schedule.

Alternatively, if marginal costs are increasing, consider symmetric generators and transmission contracts. Summing (6) over all i gives

$$\frac{dQ}{dk_j} = \frac{P_2'}{C_i''(q_i) - n(q_i + k_i)P_2'' - (n+1)\, P_2'}, \tag{9}$$

which is negative for

$$\frac{QP_2''}{-P_2'} < \left(n + 1 + \frac{C_i''(q_i)}{-P_2'}\right) \frac{Q}{Q + nk_i}. \tag{10}$$

For constant marginal costs the condition coincides with the condition for asymmetric generators (8), while the condition is less stringent than (8) for increasing marginal costs, $C_i''(q_i) > 0$.

If output decisions respond not just to transmission contract holdings, but conjectures about other generators' responses, the denominator in (4) may change in unpredictable ways without further conditions. Of these the simplest is that generators have linear marginal costs, face linear demands, and submit linear supply function bids, as in Green (1993) , specifying the quantity $q_i(p) = q_{0i} + \tau p$ they are prepared to provide at price p. In that case the denominator in (4) is constant, independent of k_i, and the analysis goes through much as before. The derivation is straightforward but slightly tedious. With linear demand and linear symmetric marginal costs, Appendix A shows that total output $\sum_i q_i$ is decreasing in the amount of transmission contracts held by generators, $\sum_i k_i$. In a supply function equilibrium generators choose the price responsiveness of their output, τ, together with $q_{0,i}$. In our model with zero uncertainty the only equilibrium will be the Nash Cournot case $\tau = 0$, already considered. The parameter τ can therefore be considered as determined by the degree of competitiveness and the extend of demand variation/uncertainty.

As generators with market power already reduce output relative to the competitive equilibrium, the reduction of output due to transmission contracts induces a further deviation from the competitive (welfare optimal) equilibrium. This result is summarised in the following Proposition, which reinforces results in J&T:

Proposition 1 *In a two-node capacity-constrained network, transmission contracts enhance the market power of importing generators in the spot market and reduce total output if demand is linear, or if either marginal costs are constant or if firms are symmetric, and the convexity of demand is less than a critical level.*

In general an oligopoly generator values the marginal transmission contract less than a trader. If a generator owns more transmission contracts, then he will withhold more output and thereby increase prices, benefiting both the trader and generator. However, unlike the trader, the generator foregoes the revenue on the marginal unit of output and therefore values the market power-enhancing transmission contract less then a trader. He will therefore be outbid in any efficient market for transmission contracts. To see this, observe that generator i's profit in the spot market is given by (2):

$$\pi_i^{spot}(q_i, k_i) = (q_i + k_i)\, P_2 - C_i(q_i) - k_i c_1,$$

so that the marginal value of additional transmission contracts is:

$$\frac{d\pi_i^{spot}(k_i)}{dk_i} = \frac{dq_i}{dk_i}\left[p_2 - C_i'(q_i)\right] + (q_i + k_i)\, P_2' \frac{dQ}{dk_i} + p_2 - c_1.$$

Substituting from (6) and using (3) gives

$$\frac{d\pi_i^{spot}(k_i)}{dk_i} = \left(p_2 - C_i'(q_i)\right) \frac{P_2' + \left((q_i + k_i)P_2'' + 2P_2' - C_i''(q_i)\right)\frac{dQ}{dk_i}}{-[P_2' - C_i''(q_i)]} + p_2 - c_1.$$

Substituting from (5) gives:

$$\frac{d\pi_i^{spot}(k_i)}{dk_i} = \left(p_2 - C_i'(q_i)\right)\left(\frac{dq_i}{dk_i} - \frac{dQ}{dk_i}\right) + p_2 - c_1. \tag{11}$$

In (11) the marginal value of a transmission contract for a generator is less than the value $p_2 - c_1$ the contract has for a trader if $\frac{dq_i}{dk_i} < \frac{dQ}{dk_i}$. That condition is satisfied for constant marginal costs $C_i'' = 0$ with a slightly enhanced convexity condition than (8):

$$\frac{QP_2''}{-P_2'} < (n-1)\frac{Q}{Q + \sum k_i - q_i - k_i}. \tag{12}$$

For symmetric generators with increasing marginal costs, (10) is slightly strengthened to

$$\frac{QP_2''}{-P_2'} < n\frac{Q}{Q+nk_i}.$$

The intuition is that a generator obtaining additional contracts will replace some production by imports: $\frac{dq_i}{dk_i} < 0$. The remaining generators face a larger market and typically increase output $\frac{dq_i}{dk_j} > 0$ for $j \neq i$. Therefore $\frac{dQ}{dk_i} - \frac{dq_i}{dk_i} = \sum_{i \neq l} \frac{dq_i}{dk_i} > 0$ and the marginal value of transmission contracts is less for a generator than for a trader.

3 Retrading Transmission Contracts

If, as was common in Europe after the restructuring of the electricity supply industry, generators inherit legacy contracts, or if generators obtain contracts in a discriminatory price auction, the previous section showed that they will reduce output relative to the situation in which they have no transmission contracts. Total welfare losses are increased as production decisions are further distorted. If, however, generators can resell transmission contracts before the energy spot market opens, then it is important to see whether efficient arbitrage again eliminates this additional inefficiency.

We start with an initial allocation of import contracts in the hands of generators at the importing node. If these generators sell the contracts, then market power is mitigated. To properly model the electricity market it is important to notice that electricity flows in opposite directions cancel each other. In our two-node network, exports from the importing node increase the total amount of transmission capacity available for imports, because only the net amount of energy transmitted is subject to the transmission constraint. A system operator could therefore issue additional import contracts for node two if the same amount of export contracts from node two are requested, provided that export contracts are explicit obligations. The trader obtaining the export contracts is then obliged to schedule a transmission from the high price (net importing) node two towards the lower price (net exporting) node and hence receive a positive payment along with the transmission contract (obligation). If such netting were allowed, then our generator at the importing node could not only sell all his import contracts, but could start buying export contracts and thereby further mitigate his market power.

Typically transmission operators are concerned that they could not recover the high costs incurred for last minute balancing of the system, if traders fail to schedule the exports. Therefore most interconnection market designs do not yet include netting, although the regulator has been studying the feasibility of netting on the Dutch-German interconnection. Generators can buy export contracts up to the amount of the transmission capacity but will not receive any payment for the contracts. We assume that the import constraint is binding and the price at the importing node is above the price at the exporting node. Given that the marginal cost of exporting is above the competitive node's price, generators will not use their export contracts and ownership of export contracts will have no effect on the output decision of generators. We therefore assume, unless otherwise stated, that generators will not buy export contracts.

We assume rational expectations so that generators and traders anticipate the outcome and price in the subsequent energy market.

3.1 Oligopoly case with one firm that owns transmission contracts

Consider an oligopoly in which only one firm, i, holds transmission contracts. Suppose there are r trading periods. Initially, firm i has k_i^r transmission contracts. In each period, firm i sells Δ^j contracts, with $\sum_{j=1}^{r} \Delta^j = k_i^r - k_i^0$. The profit from the sale of contracts and the energy spot market is:

$$\begin{aligned} \pi_i^r &= q_i(k_i^0)(p_2(k_i^0) - c_2) + \sum_{j=1}^{r} \Delta^j (p_2(k_i^0) - c_1), \\ &= q_i(k_i^0)(p_2(k_i^0) - c_2) + (k_i^r - k_i^0)(p_2(k_i^0) - c_1). \end{aligned} \tag{13}$$

The first term is the profit from spot energy sales, which depends on firm i's final holding of transmission contracts. The second term is the profit from sales of transmission contracts, which also depends on the firm's final holding.

The FOC of equation (13) with respect to k_i^0 has a unique solution $k_i^{*0} = k_i^{*0}(k_i^r)$ given by

$$k_i^{*0} + \frac{p_2(k_i^{*0}) - c_1}{\frac{\partial p_2(k_i^{*0})}{\partial k_i^0}} - q_i(k_i^{*0}) = k_i^r. \tag{14}$$

If firm i could commit to sales in each period, it would sell the optimal amount $k_i^r - k_i^{*0}$ and its profit would be independent of the timing of its sales. Without commitment, firm i can maximise its profit by selling $k_i^r - k_i^{*0}$ in the final period, $j = 1$. If it sells an amount Δ before the final period, then it will sell

$$k_i^{*0} + \frac{p_2(k_i^{*0}) - c_1}{\frac{\partial p_2(k_i^{*0})}{\partial k_i^0}} - q_i(k_i^{*0}) = k_i^r - \Delta$$

in the final period, where $k_i^{*0} = k_i^{*0}(k_i^r - \Delta)$. In general,

$$k_i^{*0}(k_i^r - \Delta) + \Delta \neq k_i^{*0}(k_i^r),$$

and therefore profits are lower than with commitment, which can be sustained by selling only in the final period. Appendix B demonstrates this for Cournot case with linear demand and constant marginal cost. Hence, if a single firm owns transmission contracts and there are $r > 1$

trading periods, the firm will sell transmission contracts only in the final trading period. This corresponds to the monopolist in Allaz and Vila (1993) not falling into the forward selling trap. The Appendix shows for the case of an n-firm oligopoly with only one firm holding transmission contracts that

$$\Delta^1 = \frac{n-1}{2n}\left(Q_{comp} + k_i^r\right).$$

Note that a monopolist ($n = 1$) will not sell any contracts. Summarizing:

Proposition 2 *If only one generator holds k transmission contracts and netting is not possible then the generator will sell $\Delta = \min(\frac{n-1}{2n}\left(Q_{comp} + k\right), k)$ contracts in the last of a finite number of trading periods with Q_{comp} being the competitive output of all generators. A monopolist will not sell any contracts.*

3.2 Oligopoly case with many firms that own transmission contracts

As before, assume there are r trading periods and let $\vec{k}^r$ be the vector of initial transmission holdings. The profit for firm i from sales of transmission contracts is

$$\pi_i^r = q_i(\vec{k}^0)(p_2(\vec{k}^0) - c_2) + (k_i^r - k_i^0)(p_2(\vec{k}^0) - c_1).$$

Firm i's profit is an increasing function of k_j^0 for $j \neq i$ because transmission contracts held by importing generators enhance market power and lead to higher prices. Conditional on k_j^0 for $j \neq i$, firm i's profit-maximizing sale of transmission contracts is given by the analog to equation (14),

$$k_i^{*0} + \frac{p_2(\vec{k}^0) - c_1}{\frac{\partial p_2(\vec{k}^0)}{\partial k_i^0}} - q_i(\vec{k}^0) = k_i^r. \tag{15}$$

However, firm i would be better off if it could force competing generators to sell fewer transmission contracts. Why then do generators sell transmission contracts that enhance their market power? A monopolist would not sell contracts, indicating that selling contracts reduces aggregate oligopolist profit. It is caused by the 'failure' of oligopolists to commit to not selling. The more opportunities are available to 'cheat' on competitors, the more contracts generators will sell and the more market power will be mitigated.

The previous section showed that if only one of many generators owns transmission contracts, then he would only sell contracts in the last retrading period. The energy spot market relevant for the output decisions of oligopolists is the local energy demand plus their holdings of import contracts. By selling some of his import contracts, a generator captures some of this market and then competes for the remaining market on an equal basis with the other generators. His total sale volume, consisting of contract sales, and contracts plus energy volume in the spot market, is increased and outweighs the effect of lower prices. Prices are reduced because generators have less incentive to withhold output and therefore total energy spot market production increases. Summarising: the generator sells his contracts in the last retrading period to induce his competitors to sell less in the energy spot market. It is not attractive to sell in earlier periods as traders would offer lower prices, anticipating further sales in the final period. As the generator is the sole seller of contracts he can commit not to sell until the last period.

If several generators own transmission contracts, then all generators will sell transmission contracts in the last trading period: They can not commit jointly not to sell, and the previous argument also applies to each generator individually: Selling contracts induces competitors to sell less in the energy spot market and thereby increases individual profits relative to individually not selling. However, now generators are also oligopolists during the earlier periods of re-trading and will no longer wait for the last retrading period to sell transmission contracts. By selling contracts in the next to last re-trading period, a generator reduces the aggregate energy and contract market his competitors face in the last re-trading period and thereby reduces the competitors' sale of contracts in the last re-trading period. This increases his sales volume and market share in the overall transmission contract and energy market and outweighs the reduction in prices due to reduced exercise of market power. Therefore a generator already starts selling transmission contracts in the next to last retrading period, and so do all his competitors. This argument equally applies to all previous trading periods.

The more trading periods exist, the more transmission contracts will be sold by generators. Assume that netting is possible, generators can start buying export contracts once they have sold all their import contracts. With increasing number of trading periods generators sell all import contracts and buy sufficient export contracts, such that they get arbitrarily close to matching all their output in the energy spot market with contracts. The Appendix provides an algebraic proof for the symmetric Cournot case with linear demand and constant marginal cost, generalising Allaz and Vila 's (1993) duopoly case. The intuition is that there is no other limit at which generators would stop selling or buying contracts, and so generators forward contract almost all output and bid almost competitively in the energy spot market. Figure 2 shows for a finite number of trading periods, e.g. six, generators will reduce their transmission contract holding in each period and then buy export contracts that mitigate market power.[3]

If netting is not permitted, then this introduces a new limit - generators can only sell their import contracts but will not obtain export contracts. The grey shaded area in Figure

[3] In GNN we show that if trading takes place in continuous time with no obvious last trading period, and if either only one generator holds contracts, or symmetric oligopolists hold the same amount of contracts but can only observe if any contracts have been sold, then there is an equilibrium in which no contracts are sold.

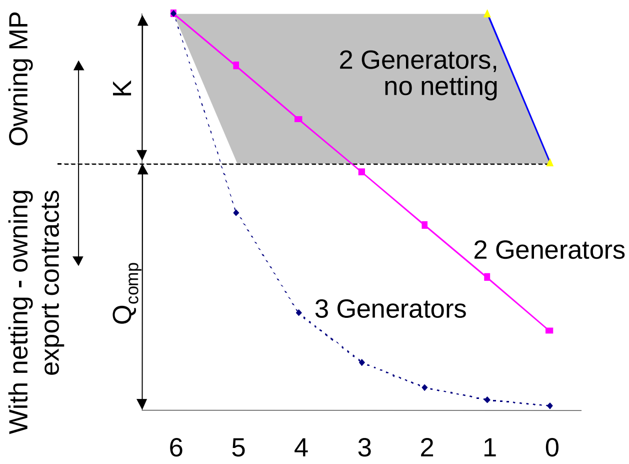

Figure 2: Contract sales by sole-owning oligopoly generator

2 illustrates that generators could start to resell contracts earlier. For our purpose it is not important when they resell. It suffices to conclude, that given a sufficient number of trading periods, generators with symmetric holding of transmission contracts will resell all market power-enhancing transmission contracts. In the energy spot market import contracts have the same influence on the profit function of a generator as does his own output sold in the energy market. It is therefore a smooth change from generators without transmission contracts to generators holding some contracts, and the limit does not create new constraints that could enable oligopolist generators to use new strategies. The results from the situation with netting apply: within a finite number r trading periods generators will sell all their import contracts. If more than r trading periods exist, then we are not interested in the sales of contracts in the initial periods. It suffices to know that eventually only r periods are left within which the generators will certainly sell all remaining contracts.

Proposition 3 *If a finite number of well-defined trading periods exists, symmetric generators will sell a finite volume of market power-enhancing transmission contracts and with sufficient trading periods they will sell all their market power-enhancing contracts. If netting is possible, then generators subsequently start buying market power mitigating transmission contracts and will in the limit forward contract all their output as the number of trading period tends to infinity.*

3.3 Continuous retrading possible

The previous paragraphs assumed that generators trade at well-defined trading events and can therefore not condition their sales on simultaneous sales of fellow generators. If instead we assume continuous trading, then at any time only one generator sells transmission contracts and so all subsequent sales can be conditioned on this sale. Furthermore continuous trading in an open interval ensures that there is always time for additional sales. We assume that the whole trading interval is short enough such that generators and traders do not discount prices during the period. This allows us to abstract from time and look only at the sequence of trading events.

Assume n generators own symmetric amounts k of transmission contracts. Any generator can determine in an open time interval whether to sell transmission contracts to a trader. A generator will either sell all his transmission contracts or keep all of his transmission contracts, because profits are increasing in the volume of contracts sold. Figure 3 shows the game structure of arbitrary, symmetric, generators selling in reaction to previous sales and in anticipation of potential subsequent sales. The game will be solved backwards.

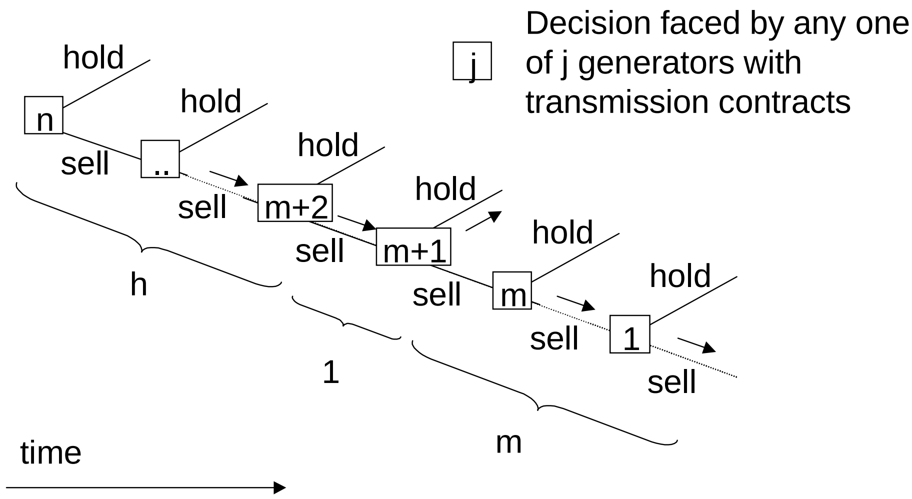

Figure 3: The game players face when re-trading continuously. At each moment only one generator decides whether to sell his contracts.

Proposition 10, in appendix C, shows that the last generator (number 1) always sells his contracts iff the previous $(n-1)$ generators already sold their contracts. Using backward induction one can show that the last m generators also sell their contracts if the previous $(n-m)$ generators have sold their contracts. If $(n-m-1)$ generators have already sold their contracts, then it is not profitable for generator $m+1$ to sell his contracts, because he only obtains the price transmission contracts obtain if no generator holds contracts. Therefore $m+1$ generators holding transmission contracts is an equilibrium outcome. Appendix C shows that $m+1$ generators typically represent 50%-60% of all generators and therefore never more than half of transmission contracts will be re-sold.

If more than $m+1$ generators hold transmission contracts, than proposition 11 shows, that it is profitable for a generator to sell transmission contracts if he is one out of $m+1+h'$, with $1 \leq h' \leq h$ generators holding contracts. He will obtain the price contracts take, if $m+1$ generators continue to hold contracts. For $n \leq 7$ generators we obtain the result that $m+h+1 \geq n$ and so it is profitable for the first $n-m-1$ generators to sell their transmission contracts, while $m+1$ generators will retain their contracts.

Starting with $n = 8$ and depending on the size of total transmission contracts held relative to total competitive output we can find situations with $m+1+h+1 = n$ when it is not profitable for the first generator to sell his transmission contracts because it will trigger h other generators to follow. In this case all transmission contracts are retained.

3.3.1 Continuous retrading with one generator owning transmission

Whilst it is possible for several generators to inherit legacy transmission contracts, the outcome of a pay-as-bid auction to be described below is that only one generator will secure contracts (the probability of two independently randomizing bidders choosing the same constant bid price for all capacity is zero). The same question arises whether this generator will resell the transmission contracts acquired in the auction.

If an oligopolist sells Δ_i of its contracts, the extra profit is:

$$\begin{aligned} \Delta\pi^{sell}_{\Delta_i} &= \pi^{spot}_{k_i-\Delta_i} - \pi^{spot}_{k_i} + \Delta_i \left(p_{2,k-\Delta} - c_1\right) \\ \Delta\pi^{sell}_{\Delta_i} &= \frac{Q_{\text{comp}} + \sum_{j=1}^{n}\left(k_j - \Delta_j\right)}{\alpha\left(n+1\right)} \left(\Delta_i - \frac{2}{n+1}\sum_{j=1}^{n}\Delta_j\right) - \frac{\left(\sum_{j=1}^{n}\Delta_j\right)^2}{\alpha\left(n+1\right)^2}. \qquad (16) \end{aligned}$$

Setting $k_j = \Delta_j = 0, \forall j \neq i$ in (16) it follows that the generator only profits from a sale ($\pi^{sell}_{\Delta_i} > 0$) if he sells fewer transmission contracts than

$$\Delta_i < \frac{n-1}{n}\left(Q_{comp} + k_i\right). \qquad (17)$$

If several trading periods exist then traders buying transmission contracts from the oligopolist fear that the oligopolist will sell additional transmission contracts in subsequent periods, reducing the value of the transmission contracts. Again, the rational expectations equilibrium with no obvious final selling period is that traders will only buy at the price these contracts take if the generator sold all of them. This argument corresponds to Admati, Pfleiderer and Zechner's (1994) analysis of takeovers: If no final trading period exists, then shareholders anticipate that a raider will continue to buy shares, thereby increasing the value of the firm. Therefore they will only sell at the value shares take, if the raider owns all shares, which might not be profitable for the raider. It is profitable for the generator to sell all transmission contracts at once $\Delta_i = k_i$ in (17) if and only if

$$k_i < (n-1)Q_{comp}.$$

This is the opposite condition to that which will be presented in (E.1) for participating in the pay-as-bid auction, so either generators will not bid and hence have nothing to sell, or will bid and not subsequently sell. This is summarised in:

Proposition 4 *If an oligopoly importer acquires transmission contracts in a pay-as-bid auction, then it is not profitable to resell any contracts if there is no final trading period.*

3.3.2 Continuous retrading with limited information flows

The previous section required that all traders and generators are instantaneously informed about how many generators already sold their transmission contracts. In a bilateral market that is certainly a strong assumption. Therefore we restrict the information flow and assume that only one signal $s\epsilon\{0,1\}$ is publicly available, which indicates whether any transmission contracts have been traded. This simplifies the strategy space of generators to the amount of transmission contracts to sell before a sale signal $\Delta_{0,i}$, and the amount of transmission contracts to sell after a sale signal $\Delta_{1,i}\,(\Delta_{0,i})$. The previous section showed that if they sell at all, then generators sell all rights at the same time, therefore $\Delta_{0,i}\epsilon\{0,k\}$ and $\Delta_{1,i}\epsilon\{0,k\}$.

One equilibrium strategy is $\Delta_{1,i}=k$ and $\Delta_{0,i}=0$. If $n-m$ other generators sell, then it is profitable to join the crowd. (Proposition 10). However, anticipating that starting to sell will trigger a general sell out, it is not profitable to sell. First, because any trader knows so, and will only pay the low price transmission contracts take if all are sold to traders, and secondly, because all other generators will sell their transmission contracts and increase output and thereby reduce profits in the energy spot market. This can be seen by setting $l=0$, $j=n$ in (55) gives $\Delta\pi_{l,j}^{s}=k\frac{(1-n)Q_{comp}-n^2k}{\alpha(n+1)^2}<0$.

Summarising, in a world with limited information there is an equilibrium in which symmetric generators with symmetric holdings of transmission contracts will not sell any of these contracts to traders.

Proposition 5 *If additional sales are always possible and generators cannot buy negative quantities of transmission contracts and if generators and traders only obtain information of whether any generator sold transmission contracts, then there is an equilibrium in which generators will not sell transmission contracts.*

4 Allocation of transmission contracts

4.1 Allocation in the Uniform Price Auction

Following the logic of solving the problem backward from energy spot market, to re-trading of transmission contracts, we finally arrive in the period of initial allocation of transmission contracts. In a single or uniform price auction with no uncertainty all bidders pay the market clearing price. The market clearing price equals the predicted price difference between markets and therefore generators cannot make profits on their transmission contracts.

Each generator submits a bid schedule defining the capacity $k_i(\eta)$ he is willing to buy at price η and so do traders, represented by their aggregate bid schedule $k_t(\eta)$. The auctioneer determines the market clearing price:

$$\eta^* = \max \eta \quad \text{satisfying} \quad \sum_i k_i(\eta) + k_t(\eta) \geq K. \tag{18}$$

4.1.1 Traders' bid schedules

We assume a perfectly contestable market with new traders entering if any arbitrage opportunity exists. Therefore traders make zero profits and pay the auction price that corresponds to the value contracts will have in the subsequent energy market. The value of these contracts is increasing with the number of contracts held by generators or decreasing in the number of contracts held by traders. In the aggregate bid schedule of traders price η is therefore decreasing in quantity k_t:

$$\eta = P_2(k_t(\eta)) - c_1.$$

4.1.2 Generators' bid schedules

Total profit of generators is given by profit in the energy market (2) less the cost for obtaining transmission contracts. Due to arbitrage the price paid for transmission contracts equals the spot price difference between the nodes and (2) simplifies to:

$$\pi_i^{contract} = P_2 q_i - C_2(q_i).$$

The first order condition with respect to k_i, is:

$$\frac{d\pi_i(k_i)}{dk_i} = \frac{dP_2}{dQ}\frac{dQ}{dk_i}q_i + \left(p_2 - C_2'\right)\frac{dq_i}{dk_i}. \tag{19}$$

The equation differs from (5) because in the energy market transmission contracts were already present and expenses sunk, whereas in (19) generators incorporate the costs of buying transmission contracts when determining the optimal holding. Substituting $(p_2 - C_2')\frac{dq_i}{dk_i}$ from FOC (3) gives

$$\frac{d\pi_i(k_i)}{dk_i} = \frac{dP_2}{dQ}\left(q_i\left(\frac{dQ}{dk_i} - \frac{dQ}{dq_i}\frac{dq_i}{dk_i}\right) - k_i\frac{dQ}{dq_i}\frac{dq_i}{dk_i}\right) = 0. \tag{20}$$

For a monopolist $\frac{dQ}{dk_i} = \frac{dQ}{dq_i}\frac{dq_i}{dk_i}$ and he will not hold any transmission contracts. To determine the outcome with oligopolists, we assume Cournot competition with $\frac{dQ}{dq_i} = 1$ and FOC (20) changes to:

$$\frac{d\pi_i(k_i)}{dk_i} = \frac{dP_2}{dQ}\left(q_i\frac{dQ}{dk_i} - (k_i + q_i)\frac{dq_i}{dk_i}\right) = 0. \tag{21}$$

According to Proposition 1 $\frac{dQ}{dk_i} < 0$, therefore oligopoly generators in equilibrium will buy a negative amount of transmission contracts, if

$$\frac{dq_i}{dk_i} < \frac{dQ}{dk_i}. \tag{22}$$

If the assumptions made in section 2 of constant marginal costs or symmetric generators apply, then (22) will be satisfied.[4] If we assume that netting is not possible so that generators cannot buy negative amount of transmission contracts, then we obtain a boundary solution with $\frac{d\pi_i(k_i)}{dk_i} < 0$ for $k_i = 0$ and oligopolists will not buy market power-enhancing transmission contracts. This result corresponds to Grossman and Hart (1980) observation, that a raider will not takeover a firm (buy contracts), if he has to pay the price which shares take due to the improved management and does therefore not capture the benefits of his activities.

If netting is feasible and oligopolists can buy a negative quantity of transmission contracts, then they will do so. A negative quantity of transmission contracts corresponds to an energy delivery in the opposite direction of the constraint. As energy flows superimpose in electricity networks a reverse flow relieves congestion and is therefore valuable. Buying a negative quantity of transmission contracts means receiving money in exchange for, in the case of physical transmission contracts, the obligation to deliver energy, or in the case of financial contracts, the obligation to pay the price difference in the energy spot markets.

If importing generators hold negative quantities of transmission contracts, corresponding to flows against the direction in which the constraint is congested, then, according to (39), they increase their output towards the competitive equilibrium output. Market power is mitigated.

[4] Condition (22) can be rewritten using (6) gives a condition for $k_i \leq 0$ to be:

$$[(q_i + k_i)P_2'' + 2P_2' - C_i''(q_i)]\frac{dQ}{dk_i} < -P_2'. \tag{23}$$

In the case of symmetric generators (9) gives:

$$\frac{(q_i + k_i)P_2'' + 2P_2' - C_i''(q_i)}{n(q_i + k_i)P_2'' + (n+1)\,P_2' - C_i''(q_i)} < 1.$$

The denominator is positive for the easily satisfied convexity condition (10). Likewise in the case of constant marginal costs (7) equation (23) is satisfied for convexity condition (12).

Netting negative flows against positive flows is therefore valuable, but as of 2002, most European transmission auctions did not offer this facility, though it was under discussion. A financial transmission contract should not face this difficulty. In the absence of financial transmission contracts and if negative physical quantities cannot be issued, then the non-negativity constraint implies that $k_i \geq 0$ and the optimal choice of importing generators is not to participate in a uniform price auction: $k_i = 0$.

The result is summarised in:

Proposition 6 *Assume transmission is sold in a uniform price auction in a two-node capacity-constrained network with no uncertainty about future equilibrium spot prices and traders perfectly arbitrage transmission prices.*

(i) With no financial contracts and no netting of transmission flows, $k_i = 0$ for all i. Arbitraging traders outbid importing generators with market power. (As suggested by J&T, p. 475)

(ii) With financial contracts and/or netting, symmetric importing generators will offer negative import capacity or sell transmission contracts.

4.2 Pay-as-bid auction

Joskow and Tirole (2000) analysed a 'pay-as-bid' auction for a monopolist generator buying transmission contracts. In Appendix D we derive the explicit solution for the monopolist case, and extend it in Appendix E to the oligopoly case. In Europe most electricity transmission auctions are uniform, but the France-England interconnector is auctioned pay-as-bid. In period one generators and traders submit sealed bid schedules specifying the quantities they are prepared to buy at different prices. The auctioneer determines the successful bids and allocates transmission capacity and the spot market for energy clears in period two. Joskow and Tirole show that a pure strategy equilibrium does not exist for this game. A monopoly generator bids with a mixed strategy.

4.2.1 Result of the pay-as-bid auction

The results are summarised in Figure 4. The monopolist bid schedule η is drawn with uniform probability over the interval ranging from the value transmission contracts take if the generator owns no contracts to the value transmission contracts take if he owns all contracts. On average the monopolist obtains half of the contracts. In the pay-as-bid auction with asymmetric information traders sometimes bid too high and therefore risk losses. These losses are compensated by the bids that are accepted below the value of the transmission contract. In expectation traders

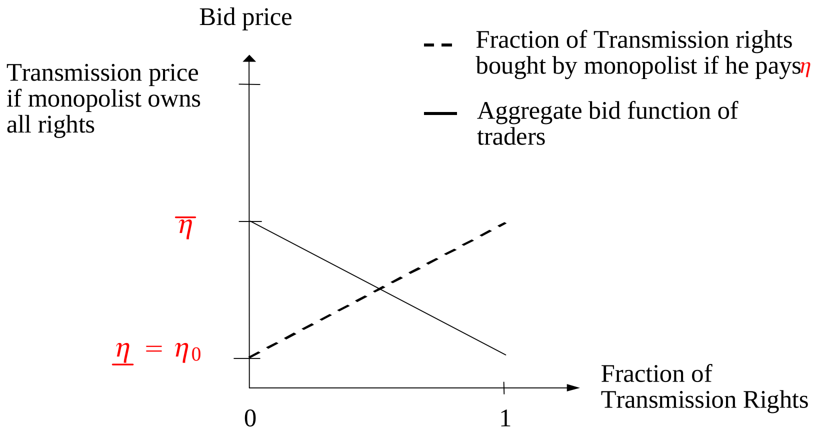

Figure 4: Bids and outcomes in pay-as-bid auction

therefore make zero profit. The marginal bid does not arbitrage prices in a pay-as-bid auction, so the monopolist obtains transmission contracts below its market value to compensate for the lost revenues on additional withheld output.

In the monopoly case the generator does not profit from obtaining transmission contracts because traders increase their bids until the generator's profits when obtaining transmission contracts equal profits when he does not obtain transmission contracts. Appendix D shows that in the oligopoly case, generators profit in expectation from the auction of transmission contracts. This is because generators' profits are increased relative to a situation without transmission contracts if other generators obtain transmission contracts in the auction and reduce output. Traders bid such that generators are indifferent between participating and not participating in the auction.

The analysis so far ignores the possibility of retrading. It has no effect in the monopoly case, because the monopolist does not sell transmission contracts. In an oligopoly with retrading in a finite number of trading periods, the single successful generator buying k rights in the auction will sell $\Delta = \min(k, \frac{n-1}{2n}(Q_{comp} + k))$ rights in the subsequent market. That affects the value of contracts in the auction and it is no longer possible to derive algebraic solutions for the auction bids. It is possible to numerically integrate the differential equations, without and with subsequent retrading, given specific parameter values, as discussed in Appendix E. Figure 5 illustrates the results of the mixed strategy equilibrium for parameter values $n = 2$, $\alpha = 1$, $K = 3$ and $Q_{comp} = 3/2$.

Generators profit from reselling transmission contracts at a price above their successful bid. This decreases the minimum profitable bid volume and increases the benefits of winning relative to free-riding, with the increased market power exercised by the winning generator. As a result,

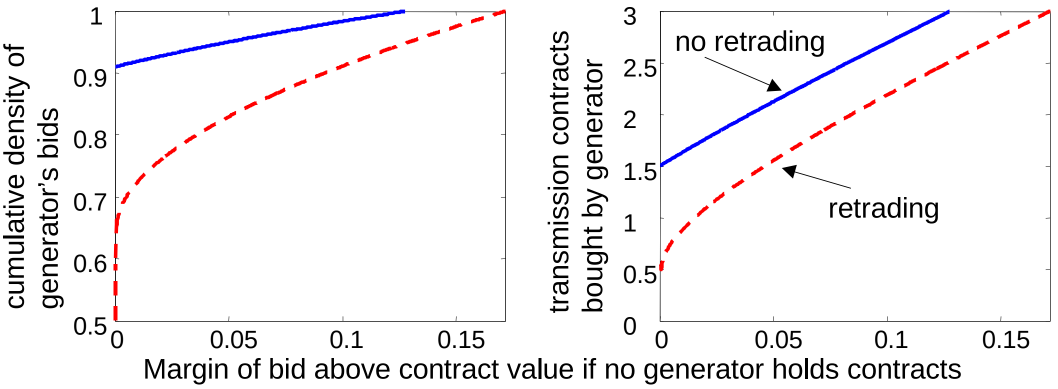

Figure 5: Bid choice and outcomes in pay-as-bid auction with retrading

generators bid more frequently in the discriminatory auction. With re-trading the generator buys contracts more frequently in the discriminatory auction (the dashed line of the cumulative density function is lower), and so traders can also bid more aggressively and the generator obtains fewer transmission contracts at a given price (see right part of figure). Allowing re-trading after a discriminatory auction is not necessarily welfare improving as it changes the bids in the auction. In our specific example expected monopoly rents from contracts obtained in the discriminatory auction increase from 0.12 units to 0.27 units if retrading is possible. Furthermore efficiency is reduced: the expected holding of transmission contracts by generators at the time of the energy spot market (after potential retrading) increases from 0.38 to 0.51 (i.e. from $0.21Q_{comp}$ to $0.34Q_{comp}$). This translates directly into withholding of more output and hence more inefficient production decisions. In the example, the higher participation rates outweigh the benefits from reselling market power-enhancing contracts during retrading.

We summarize these results in the following proposition which generalises J&T monopoly case (J&T p. 461-462).

Proposition 7 *Assume constant marginal costs and linear demand. If transmission is sold in a pay-as-bid auction in a two-node capacity-constrained network where import capacity exceeds* $(n-1)$ *times domestic competitive output, then importing generators with market power will play a mixed strategy and secure some fraction of transmission contracts, enhancing their market power.*

5 Exporter market power in a two-node network

We briefly consider the case in which the oligopoly is located at the exporting node of a two-node network (Figure 6). The importing node is assumed perfectly competitive with price $p_2 = c_2$.

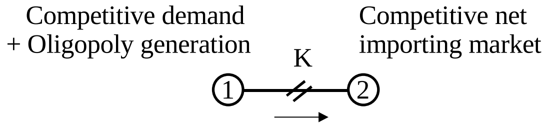

Figure 6: Oligopoly exporting case

The analysis parallels section 2, but as the direction of trade is exactly opposite, so too are the results. The counterparts to (1) and (2) are

$$\begin{aligned} p_1 &= P_1(Q-K), \qquad p_{1,comp} < c_2 = p_2, \\ \pi_i(q_i,k_i) &= P_1 q_i - C_i(q_i) + k_i(c_2 - P_1). \end{aligned} \tag{24}$$

Similarly, the counterpart to (6) is

$$\frac{dq_i}{dk_j} = \frac{-\delta_{ij}P_1 + [(q_i-k_i)P_1'' + P_1']\frac{dQ}{dk_j}}{-[P_1' - C_i''(q_i)]}. \tag{25}$$

The same reasoning as in the importing case establishes that $dQ/dk_i > 0$, so transmission contracts mitigate, rather than enhance, market power. This time exporting generators will successfully bid against arbitrageurs, except in the monopoly case, as the next section shows.

5.1 Uniform price auction

Consider the case in which traders perfectly arbitrage the auction ensuring that $\pi_i^{contract}(k_i) = P_1 q_i - C_i(q_i)$. The counterpart to (20) is

$$\frac{d\pi_i^{spot}(k_i)}{dk_i} = \frac{dP_1}{dQ}\left(q_i\left(\frac{dQ}{dk_i} - \frac{dQ}{dq_i}\frac{dq_i}{dk_i}\right) + k_i\frac{dQ}{dq_i}\frac{dq_i}{dk_i}\right) = 0. \tag{26}$$

Again, for the same reason as in 4.1.2, a monopolist will not buy any contracts at the arbitraged price. In the oligopoly case, (26) can be solved for k_i :

$$0 < k_i = q_i\left(1 - \frac{dQ/dk_i}{\frac{dQ}{dq_i}\frac{dq_i}{dk_i}}\right) < q_i,$$

provided that no more transmission contracts are demanded than are available, $\sum_i k_i \leq K$, otherwise generators outbid all traders, and the auction clearing price η will be such that $\sum_i k_i = K$. Thus for the case of linear demand $D = A = \alpha P_1$, the optimal contract volume k_i for each of the n symmetric exporting oligopolistic generators is

$$k_i = Min\left(\frac{n-1}{n^2+1}(A+K-\alpha c_1), \frac{K}{n}\right) = Min\left(\frac{n^2-1}{n^2+1}q_{i,k=0}, \frac{K}{n}\right). \tag{27}$$

In a single price auction, exporting oligopoly generators (but not monopolists) secure some or all of the available transmission contracts. As their spot energy output is increasing in their contract position ($dQ/dk_i > 0$) so transmission contracts mitigate, rather than enhance, market power. These contracts mitigate market power by driving down the price at the export node below that prevailing if generators are prevented from acquiring transmission contracts, while leaving the price at node two unaffected (as the link is fully used in any case). Allowing generators to participate in the auction and even foreclose the market is welfare improving. Transmission contracts towards node two correspond to contracts for differences and so generators use the transmission contracts to sell energy in the forward market. This corresponds to a second contracting stage and makes the outcome more competitive, as in Newbery (1998). In that model, long-term contracts are signed in period one, and the remaining energy is traded in the spot market in period two. The more energy is covered by long-term contracts, the lower the exposure of generators to the spot-market price. Therefore they increase their output to obtain revenue on the marginal unit at the cost of overall lower prices. Rational expectations imply that the expected spot-market prices feed back to the long-term contract prices. The result shows that allowing access to the transmission line decreases market power by serving as an initial contracting stage. This general result confirms Stoft's (1999) special case for zero demand elasticity at the import node and the J&T (p. 472) result that price in the exporting region is reduced if a monopoly generator holds transmission contracts.

Provided there is complete information about costs and demand and no uncertainty, the pay-as-bid auction has the same equilibrium, because the oligopolistic generators use a pure strategy. We summarise the result as:

Proposition 8 *In a two-node capacity-constrained network, allowing exporting generators with market power to buy transmission contracts is always welfare-enhancing.*

This result fails for meshed networks (see Section 6.2).

5.2 Retrading transmission contracts

The equilibrium just computed was for a one-shot transmission auction. The fact that transmission contracts act as a commitment device raises the question whether it might be profitable for generators to increase their holding of transmission contracts after the initial auction (or buy from those who hold contracts if there is no auction). As in Section 3 the change in profits if a generator buys an additional Δ_i transmission contracts is:

$$\begin{aligned} \Delta\pi_i^{\text{buy}} &= \pi_{k_i+\Delta_i}^{spot} - \pi_{k_i}^{spot} - \Delta_i \left(c_2 - p_{1,k_i+\Delta_i}\right), \\ &= \frac{\Delta_i}{n+1}\left((n-1)\left(p_{1,k_i} - c_1\right) - \frac{n}{\alpha}\frac{\Delta_i}{n+1}\right). \end{aligned} \tag{28}$$

Buying additional transmission contracts is unprofitable for a monopolist ($n = 1$), because he has to pay the higher ex-post price. However, in an oligopoly setting ($n > 1$) buying additional small quantities of contracts $\Delta_i < \frac{\alpha}{n}\left(n^2 - 1\right)(p_{1,k_i} - c_1)$ is profitable for all generators as long as the price-cost margin $p_{1,k_i} - c_1$ is positive. Oligopolists have the advantage that by buying contracts they commit to higher output, inducing competitors to reduce their output. The reduction of competitors' output has a positive impact on profits, which is not present in the monopoly case.

If several trading periods exist, then traders anticipate that oligopolists will buy additional transmission contracts in subsequent periods. The value of transmission contracts rises with the total amount held by generators and traders therefore charge a high price starting with the first sale. Consider the change of profits for a generator buying Δ_i contracts when the remaining contracts are bought by other generators and when traders charge the price corresponding to all capacity held by generators:

$$\Delta\pi_{\text{buy } \Delta_i \text{ at final price}} = \frac{1}{\alpha\,(n+1)^2}\left[K(K - 2Q_{comp}) + (n+1)\,\Delta_i(Q_{comp} - K)\right]. \tag{29}$$

If $K < Q_{comp}$ then profit increases in the amount of transmission contracts a generator buys and he will therefore buy as many contracts as possible. Participation of generators is guaranteed, because if any generator obtains a large enough fraction of all contracts, then his profits are higher than in a no-trade situation. As a result generators buy all contracts from traders, even though their aggregate profit is reduced: $\sum \Delta\pi = -K\left((n-1)\,Q_{comp} + K^2\right) / \left(\alpha\,(n+1)^2\right)$.

The situation is similar to a Coasian durable goods monopolist. Generators find themselves in the unenviable position of buying additional transmission contracts that mitigate their market power (similar to the Coasian durable goods monopolist selling to traders thereby reducing his exposure to spot prices and the incentive to exercise market power by withholding output). The only equilibrium will be that the generators buy all available transmission contracts at the price difference that corresponds to their holding all contracts, even in the original auction. If they attempt to pay less in that auction, traders will anticipate profitable sales in the aftermarket and will bid up the price to the final equilibrium price. Generators are therefore forced to commit to the higher output associated with selling all transmission output forward. This outcome corresponds to Allaz and Vila's (1993) result for $n \leq 2$ that with additional periods of trading of forward contracts the equilibrium outcome in the energy market gets closer to the competitive outcome.

For $K > Q_{comp}$ a generator's profit (29) decreases with the amount of transmission contracts Δ_i he buys. Therefore no generator buys contracts at the ex-post price and traders have to offer lower prices. Traders still benefit relative to a no-trade situation, because they capture some of the profits (28) from generators. If ownership of contracts is dispersed, then each trader waits for other traders to sell their contracts expecting to subsequently receive higher prices, but this deadlock prevents all trades. If ownership of transmission contracts is sufficiently concentrated then traders sell all contracts to generators.

The example shows that allowing generators with market power to secure transmission export contracts mitigates market power in a two-node network. As prices at the importing node are fixed at full utilisation of transmission, the transmission market acts just like a long-term contract market and induces generators to sell more energy than in a one-stage Cournot model.

6 Market power in a three-node network

Joskow and Tirole (2000) conclude in their analysis of the three-node network that "the effects of transmission contracts holding on market power on a three-node network are conceptually similar to those on a two-node network". This is no longer the case when assessing policy options to mitigate market power, as the two-node network allows a very simple characterisation of policy on transmission contracts to mitigate market power. Exporting generators should be encouraged to trade for transmission (as often as possible) while importing generators should be discouraged from obtaining transmission. The reason for the simplicity is that the market price in a competitive region connected to an oligopolistic region by a constrained transmission link is independent of who secures rights to that transmission link, so that the entire impact of generator transmission contracts is in the region with the market power.

Matters are more complex in meshed networks where single link constraints do not necessarily isolate other competitive markets from market power at a node. Section 6.3 shows that a uniform auction for transmission contracts in the presence of complete information still ensures that transmission contracts cannot enhance market power. If, however, transmission contracts are inherited or secured in a 'pay-as-bid' auction (Section 6.4), then the policy conclusion from the two-node network that only exporting generators should hold transmission contracts is no longer sufficient to prevent the enhancement of market power. In Section 6.2 we show that if the fraction of generator controlled supply sold into the oligopolistic market is increased by transmission contracts, market power is enhanced, and vice versa.

6.1 Loop flow considerations

In a simple two-node network with a single link all power from one node must flow along the single link to the other node. In a meshed network with more than one possible path from one node to another, electricity will flow over all links, distributed according to Kirchoff's Laws (Bohn et.al. 1984). Thus in Figure 7, a generator at node two may sign a contract to deliver power to a consumer at node three, and then seek to sign a contract with the transmission operator of the most direct link, $\overline{23}$, but only some of the power will actually flow along this link, with the balance creating 'loop flows' along all other paths connecting the source (the generator) to the sink (final consumer), in this case along $\overline{21}$ and $\overline{13}$. Dealing with these loop flows bedevils the management of interconnected transmission systems, in which various sub-grids of the interconnected system are under the jurisdiction of separate Transmission System Operators (TSOs). One direct consequence of these loop flows is that a transmission constraint on one link impacts on the flows that are possible on every electricity transmission link in the network. Two different approaches have been proposed to explicitly address transmission constraints and allocate scarce transmission capacity in a liberalised electricity market: property rights and nodal prices.

First, property rights allocate physical transmission capacity either on a constrained link (flow-gate rights), or for transmission between two locations (point-to-point contracts) or insertion and withdrawal at specified locations (entry/exit rights). Flow-gate rights require that any energy trade is matched with individual property rights for each transmission constraint in the network, and seem therefore infeasible in most real networks (Hogan, 2000). Point-to-point contracts and entry/exit rights aggregate the underlying information to increase liquidity and facilitate trading, but require a central system operator to define the aggregated rights based on the fundamental flow-gate rights. These difficulties of aggregation are not addressed in this paper, therefore it suffices to analyse flow-gate rights - the results also apply to point-to-point contracts and entry/exit rights. In the flow-gate design the system operator calculates propor-

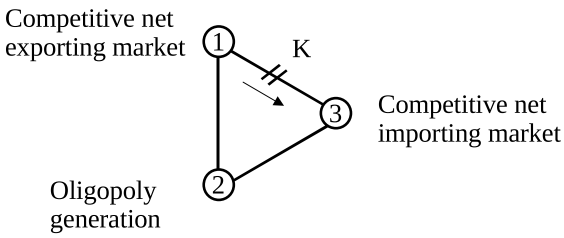

Figure 7: Symmetric 3-node network with a single constraint

tionality factors γ_{ij}^{k} to determine what proportion of energy flow between injection node i and offtake node j will pass over link k. The proportionality factor γ_{ij}^{k} is negative if the energy flow goes in the opposite direction to the defined orientation of the link.[5] A trader m multiplies the power volumes q_{ij}^{m} (positive amount of MW)[6] he wants to transmit between different nodes with

[5]The orientations are determined arbitrarily, and these will determine the signs of the factors γ_{ij}^{k} and hence the consistency of the flow analysis.

[6]Energy is measured in MWh, while capacity is measured in MW. We define a unit of time during which flows

the corresponding proportionality factor γ_{ij}^k. This determines how many flow-gate rights he has to obtain for each link $f^{k,m} = \gamma_{ij}^k q_{ij}^m$. The system operator can issue or auction (O'Neill et.al. 2000) a net amount of flow-gate rights $\sum_m f^{k,m}$ up to the capacity K^k of the link and market participants can subsequently trade these flow-gate rights.

In the second approach, nodal pricing, generators, traders and consumers submit energy bids to a system operator, who is the central auctioneer. Each bid specifies a location in the network, a quantity of energy to be offered or requested and a price. The system operator determines, de facto simulating a market for flow-gate rights, the market clearing price at each node. Generators receive the nodal price of their injection point while consumers pay the nodal price at the off-take point. Nodal pricing can be interpreted as an interface to simplify the underlying market structure and reduce transaction costs to match physical transmission contracts to energy delivery. If generators and consumers sign long-term energy contracts, then nodal pricing alone would expose them to uncertainty about the difference in nodal prices between their locations. The system operator can issue financial transmission contracts (FTCs), which pay the price difference between two locations for a specified quantity of energy, thus facilitating long-term contracting and providing a market signal for future transmission demand. The advantage of FTCs over physical transmission contracts is that they can serve their hedging purpose even if aggregated over larger areas and time periods - thereby increasing liquidity and reducing transaction costs. Nodal pricing also exposes generators to more net-demand responsiveness, thereby reducing their incentive to exercise market power (Neuhoff, 2003). For present purposes these differences are ignored. If we assume that energy spot markets are perfectly liquid and allow for continuous trading while physical transmission contracts can be easily reconfigured, then Joskow and Tirole (2000) show that financial transmission contracts provide the same financial incentives to generators as do physical transmission contracts. Therefore the results for physical flow-gate rights will directly translate to financial transmission contracts in a nodal pricing design.

6.2 Exporting generator holding transmission contracts

The following example demonstrates that in meshed networks exporting oligopolists may use transmission contracts to enhance their market power. As before, the game is solved backwards, first determining equilibrium prices in the energy market, and then determining the bidding strategy of generators and traders in stage one. As in Joskow and Tirole (2000), a competitive net exporting market is located at node one, an oligopoly at node two and competitive net demand at node three (Figure 7).

As described in the previous section, market participants have to obtain flow-gate rights for

are constant, and the energy is then MW multipled by the time interval, taken here as 1 unit.

the constrained link $\overline{13}$, if they want to transmit energy between any two nodes in the network. Due to loop flows $\gamma_{13} = 2/3$ of electricity exports Q_1 from node one and $\gamma_{23} = 1/3$ of exports Q_2 from node two pass along the constrained link $\overline{13}$. To sell one unit of energy from node two to three, an oligopoly exporter therefore needs 1/3 of a unit of flow-gate on $\overline{13}$. To sell one unit of energy from node one to node three, 2/3 of a unit of flow-gate rights on $\overline{13}$ is required. In this particular network total exports are therefore constrained by:

$$2Q_1 + Q_2 \leq 3K. \tag{30}$$

The value η of transmission contracts to the flow-gate is proportional to the price difference between nodes times the inverse of the proportion of flows between the nodes that goes along the flow-gate, and this value defines the efficient arbitrage condition:

$$\eta = \frac{3}{2}(p_3 - p_1) = 3(p_3 - p_2). \tag{31}$$

Assuming competitive constant marginal cost c_1 at node one and linear demand $p_3 = A - Q_1 - Q_2$ at node three with intercept A and using binding constraint (30), (31) can be solved for p_2 as a function of p_1 and p_3:

$$p_3 = \frac{2A - 3K - Q_2}{2}; \qquad p_2 = \frac{c_1}{2} + \frac{2A - 3K - Q_2}{4}. \tag{32}$$

In the energy market oligopolist i at node 2 who produces q_i with constant marginal costs c_2 and who owns k_i transmission contracts (flow-gate rights on $\overline{13}$, each of which allows him to sell 3 units from node 2 to 3) maximises the following profit function:

$$\pi_i^{spot} = (p_2 - c_2)\, q_i + 3\,(p_3 - p_2)\, k_i. \tag{33}$$

Substituting p_2 from (32) in (33) the FOC gives the optimal output choice:

$$\begin{aligned} q_i &= \frac{2c_1 - 4c_2 + 2A - 3K + \sum_j 3k_j}{n+1} - 3k_i\ ; \\ \pi^{spot}(k_i) &= \frac{\left(Q_{comp} + \sum_j 3k_j\right)^2}{4\,(n+1)^2} + 3k_i\,(c_2 - c_1)\,; \\ Q_2 &= \frac{n\,(2c_1 - 4c_2 + 2A - 3K) - \sum_j 3k_j}{n+1}. \end{aligned} \tag{34}$$

The competitive output would be $Q_{comp} = Lim_{n \to \infty} Q_2 = 2c_1 - 4c_2 + 2A - 3K$. Equation (34) shows that ownership of transmission contracts k_i causes the oligopolist to decrease output q_i. To evaluate the effect of transmission contracts we can apply the First Welfare theorem according to which the competitive equilibrium is efficient. The output in a situation with market power and without transmission contracts is reduced relative to the competitive output by the factor $\frac{n}{n+1}$.

If generators own transmission contracts, then their market power is enhanced and they reduce the output by an additional $\frac{1}{n+1}\sum_j 3k_j$. As all other nodes are competitive, increased deviation from the efficient level at the distorted node increases inefficiency. It follows, as J&T already showed for the monopoly case, that holding transmission contracts by the exporter can decreases welfare. Whereas in the two-node network only transmission contracts held corresponding to imports enhance market power, in the meshed network even transmission contracts corresponding to exports can enhance market power.

6.3 Effect of single price auction

The next step is to see whether there is any danger of the exporter acquiring market power enhancing contracts in an auction. As before, there are no problems with a uniform price auction, which will reveal all information and therefore result in arbitrage of prices. Generators chose the proportion of transmission contracts to buy k_i in order to maximise total profits $\pi_i^{auction} = \pi^{spot}(k_i) - 3k_i\,(p_3 - p_2)$. The first order condition gives the total quantity to be obtained:

$$3k_i = \frac{1-n}{1+n^2} Q_{comp} \leq 0.$$

In a uniform price auction the oligopolist buys in equilibrium a negative quantity of transmission contracts, therefore market power is mitigated to the same degree as calculated for the two-node network in section (5.1). Note that this requires the System Operator to ensure that the oligopolists indeed acts to relieve the constraint, and allows additional flow-gate rights to be issued to other participants (the arithmetical or directional sum over all contracts is the binding constraint). If negative quantities of flow-gate rights are not available, then generators will buy no transmission contracts and the result that market power is not enhanced if transmission contracts are auctioned in a uniform price auction survives. The following theorem summarising these more general results is proved in appendix (F).

Theorem 9 *If constrained transmission capacity in a meshed network is sold in a single price auction that is efficiently arbitraged by traders who can accurately predict future equilibrium spot prices, then oligopolists will only acquire contracts that mitigate market power. Marginal costs of generation should not increase by more than* $\left(\sqrt{2}-1\right)$ *times demand slope. If generators are asymmetric, then marginal costs of each generator should not increase by more than* $1/n$ *times demand slope (lower bounds due to approximations).*

These conditions should be easily satisfied, because electricity demand is very inelastic (demand slope high) while marginal costs of generators that can alter their output decision are comparatively flat.

If transmission contracts are formulated as options and not as obligations, then generators cannot acquire a negative quantity of transmission contracts. If negative quantities would mitigate market power, then generators will end up not acquiring any transmission contracts. This is the reason for the weak formulation in theorem (9) that transmission contracts will not enhance market power. If transmission contracts are formulated as obligations than they will mitigate market power if allocated in a uniform price auction with full information.

6.4 Pay-as-bid auction

If oligopolists only buy negative transmission contracts in a single price auction, would a pay-as-bid auction allow them to secure transmission and enhance market power by playing a mixed strategy, as in the two-node example? As before, the first step is to see if an oligopolist would increase profits by buying transmission contracts at the value the contracts have if traders do not expect oligopolists to obtain contracts:

$$\begin{aligned}\Delta\pi_k^{auction} &= \pi_{k_i}^{spot} - \pi_{k_i=0}^{spot} - 3k_i\left(p_{3,k=0} - p_{2,k=0}\right),\\ &= \frac{1}{4\alpha}\frac{3k_i}{(n+1)^2}\left(3k_i - (n-1)\,Q_{comp}\right).\end{aligned}$$

As in the two-node case (E.1), the ratio between transmission capacity $3k_i$ and $(n-1)\,Q_{comp}$ determines whether generators can profitably bid for market power-enhancing contracts when information asymmetry prevents perfect arbitrage. However, in the three-node case oligopolists $(n > 1)$ are more likely to play a mixed strategy equilibrium than in the two node case. This is because in the meshed network export capacity from node two is three times the transmission capacity of the line $\overline{13}$. It is reflected in the factor $3k_i$ instead of k_i in the brackets. Mixed strategy equilibria with their market power enhancing implications are therefore more likely in meshed networks than in the two node case. In appendix (G) the mixed strategy equilibrium of a monopolist in the three node network is calculated analogously to the situation in a two node network.

6.5 Selling inherited contracts

Finally, it is important to see whether inherited transmission contracts enhance market power or these additional distortions can be traded away. The oligopolist's total profit function is

$$\begin{aligned}\Delta\pi_{\Delta k_i}^{sell} &= \pi_{k_i-\Delta k_i}^{spot} - \pi_{k_i}^{spot} + 3\Delta k_i\left(p_{3,k_i-\Delta k_i} - p_{2k_i-\Delta k_i}\right),\\ &= \frac{3\Delta k_i}{(n+1)^2}\left(\left(n^2-1\right)\left(p_{2,k_i} - c_2\right) - \frac{n}{4\alpha}3\Delta k_i\right). \qquad (35)\end{aligned}$$

The situation corresponds exactly to the two node network with an importing oligopolist inheriting transmission contracts. The oligopolist could profitably sell small quantities of transmission contracts at the ex-post price. However, traders will not buy contracts as they anticipate that further sales by oligopolists would further reduce the value of their contracts and we return to the same discussion we presented for the two node network.

6.6 Conclusion meshed Networks

We showed that in arbitrary meshed networks oligopoly generators can obtain market power enhancing contracts in a discriminatory price auction, whereas a perfectly arbitraged uniform price auction with complete information only allocates market-power-mitigating contracts. The intuition about auctions therefore does not differ from the two-node network. If the requirement of complete information and perfect arbitrage of the uniform price auction is not satisfied, then the policy suggested for the two-node network was to restrict generators to hold export transmission contracts. In meshed networks this policy no longer suffices, because even export contracts from the generation node can enhance market power if the price at the destination reacts strongly to output changes of the generator. One solution is to define transmission contracts to a reference node that has a price least influenced by any generator's output decision, e.g. node one in our example. Generators should then be restricted to transmission contracts with this reference node. Consumers and their representatives should then obtain transmission contracts from the reference node onward. Such a policy minimises the risk that transmission contracts enhance market power while ensuring that they provide risk hedging services and provide information for network expansion.

7 Conclusions

Allowing generators with market power access to transmission auctions when transmission capacity is constrained may amplify or mitigate their market power. Regulators may therefore wish to consider under what circumstances it would be desirable to prevent such generators from securing or retaining transmission contracts. We find that in the two-node case if the generators with market power are located at an import constrained node, it is always undesirable to allow them to retain existing transmission contracts. If they are to be compensated, it should be by a formula that does not depend on the subsequently realised spot price, to make sure that they have no additional reasons for influencing that price. It is also undesirable to allow them access to the transmission auction where this is pay-as-bid, or where there is likely to be asymmetric information favouring the generators. Even under the ideal circumstances of perfectly informed arbitrageurs and a single-price auction, while there may be no harm in allowing generators to bid, there is also no benefit. Nor do the generators benefit from bidding even in the pay-as-bid case, as all the additional distortionary revenue is secured by the transmission company.[7] On the other hand, if the generators with market power are located at an export constrained node, their bids for transmission capacity allow them to pre-commit to additional output and reduce prices at the exporting node, to the benefit of consumers there.

In a three-node network (and with more nodes), transmission contracts can enhance market power even when they correspond to own production (and not just imports), at least if loop flows imply that output changes have a bigger impact on the price at the delivery node than at the origin node. One solution is to define transmission contracts to the reference node that has a price least influenced by any generator's output decision. Generators should then be restricted to transmission contracts with this reference node. Consumers and their representatives should then obtain transmission contracts from the reference node onward. Such a policy minimises the risk that transmission contracts enhance market power while ensuring that they provide risk hedging services and provide information for network expansion.

We analysed the effect of transmission contracts on output decisions when transmission constraints are permanently binding. However, links between several regions are only constrained part of the time with intermediate periods where the constraint is 'just' binding or 'just' not binding. While there are several models showing that market power can increase the period when constraints are binding, the effect of transmission contracts on whether or not constraints are binding is still an open question.

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[7] In some European countries, generators still have an ownership interest in transmission, and this would provide an additional motive for market manipulation, even with "functional" unbundling and separation agreements (as required by the EU Electricity Directive). This may provide additional reasons for banning the acquisition of such transmission rights.

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A Derivation of dependence of $\sum q_i$ on $\sum k_i$ with conjectured supply function equilibria

If demand $D = A - \alpha p_2$, then the residual demand facing generator i is

$$q_i^r = A - \sum_{j \neq i} q_j(p_2) - K - \alpha p_2 = A - \sum_{j \neq i} q_{0,j} - K - (\alpha + (n-1)\tau)\, p_2. \tag{36}$$

If costs $C(q) = c_2 q + \frac{\beta}{2} q^2$ (linear marginal costs with slope β), profit is

$$\pi_i^{spot}(p_2, k_i) = q_i^r (p_2 - c_2 - \frac{\beta}{2} q_i^r) + k_i\,(p_2 - c_1)\,.$$

Using the FOC with respect to p_2 gives:

$$0 = (1 + \beta\,(\alpha + (n-1)\tau)) \left(A - \sum_{j \neq i} q_{0,j} - K - (\alpha + (n-1)\tau)\, p_2 \right) - (p_2 - c_2)\,(\alpha + (n-1)\tau) + k_i,$$

Sum over all i:

$$\begin{aligned} 0 \;=\;& (1 + \beta\,(\alpha + (n-1)\tau)) \Big(n\,(A - K) - (n-1) \sum q_{0,j} - n\,(\alpha + (n-1)\tau)\, p_2 \Big) \\ & - n\,(p_2 - c_2)\,(\alpha + (n-1)\tau) + \sum k_i, \end{aligned} \tag{37}$$

and using (36) gives the equilibrium aggregate output:

$$\begin{aligned} \sum_i q_i \;=\;& \frac{\left(1 + \frac{\tau}{\alpha}\left(n - 2 - \alpha\beta\left(1 + \frac{\tau}{\alpha}(n-1)\right)\right)\right) n\,(A - K)}{(1+n) + \alpha\beta + \frac{\tau}{\alpha}(n + \alpha\beta)(n-1)} \\ & - \frac{\left(n + (2n-1)\, n\frac{\tau}{\alpha} + (n-1)\, n^2 \frac{\tau}{\alpha}^2\right) \alpha c_2 + \left(1 + n\frac{\tau}{\alpha}\right) \sum_i k_i}{(1+n) + \alpha\beta + \frac{\tau}{\alpha}(n + \alpha\beta)(n-1)}. \end{aligned} \tag{38}$$

The parameters are positive, hence total output $\sum_i q_i$ is decreasing in the amount of transmission contracts held by generators, $\sum_i k_i$.

B Cournot case with retrading

Assume constant marginal costs, linear demand and Cournot competition ($\beta = 0$, $\tau = 0$). From (36) and (38):

$$
\begin{aligned}
q_i &= \frac{A - K - \alpha c_2 + \sum_{j=1}^{n} k_j}{n+1} - k_i, \quad Q = \frac{n\left(A - K - \alpha c_2\right) - \sum_{j=1}^{n} k_j}{n+1}, \quad \left|\sum_{j=1}^{n} k_j\right| \leq K, \\
p_2 &= \frac{A - K + n\alpha c_2 + \sum_{j=1}^{n} k_j}{\alpha\left(n+1\right)}, \quad Q_{comp} = A - K - \alpha c_2. \qquad (39)
\end{aligned}
$$

Assume that a finite number R of well-defined trading periods exists. In retrading period r, counting backward from the last retrading period 1, generator i sells Δ_i^r of his current holding of transmission contracts k_i^r. In subsequent periods each generator will sell additional transmission contracts $\sum_{j=1}^{r-1} \Delta_i^{r-j} = \Theta_i^{r-1}\left(\overrightarrow{k^{r-1}}\right)$, where Θ_i^{r-1} is a function of the vector of all contract $\overrightarrow{k^{r-1}} = \overrightarrow{k^r} - \overrightarrow{\Delta^r}$. The profit function generator i maximises in period r is therefore given by the amount of transmission contracts sold in the subsequent periods at their ex-post market value, plus the profits to be made in the energy market given the remaining transmission contracts. As traders are rational and pay the expected price difference between the energy markets for transmission contracts, the only influence of transmission contracts is via the pre-commitment effect it has on the output choice q_i and therefore also on the spot market price p_2:

$$
\begin{aligned}
\pi_i^r\left(\overrightarrow{k^r}, \overrightarrow{\Delta^r}\right) &= \left(p_2\left[\overrightarrow{k^r} - \overrightarrow{\Delta^r} - \overrightarrow{\Theta^{r-1}}\left(\overrightarrow{k^r} - \overrightarrow{\Delta^r}\right)\right] - c_2\right) q_i\left[\overrightarrow{k^r} - \overrightarrow{\Delta^r} - \overrightarrow{\Theta^{r-1}}\left(\overrightarrow{k^r} - \overrightarrow{\Delta^r}\right)\right] \\
&\quad + \left(p_2\left[\overrightarrow{k^r} - \overrightarrow{\Delta^r} - \overrightarrow{\Theta^{r-1}}\left(\overrightarrow{k^r} - \overrightarrow{\Delta^r}\right)\right] - c_1\right) k_i,
\end{aligned}
$$

Substituting energy spot price p_2 and quantities q_i from (39) gives

$$
\begin{aligned}
\pi_i^r\left(\overrightarrow{k^r}, \overrightarrow{\Delta^r}\right) &= \left(c_2 - c_1\right) k_i^r + \frac{Q_{comp} + \sum_{j=1}^{n}\left(k_j^r - \Delta_j^r - \Theta_j^{r-1}\left(\overrightarrow{k^r} - \overrightarrow{\Delta^r}\right)\right)}{\alpha\left(n+1\right)} \\
&\quad * \left(\frac{Q_{comp} + \sum_{j=1}^{n}\left(k_j^r - \Delta_j^r - \Theta_j^{r-1}\left(\overrightarrow{k^r} - \overrightarrow{\Delta^r}\right)\right)}{n+1} + \Delta_i^r + \Theta_i^{r-1}\left(\overrightarrow{k^r} - \overrightarrow{\Delta^r}\right)\right),
\end{aligned}
$$

and therefore the optimal amount of transmission contracts to be sold is given by the FOC with respect to Δ_i^r:

$$\begin{aligned}0 &= \frac{Q_{comp}+\sum_{j=1}^{n}\left(k_j^r-\Delta_j^r-\Theta_j^{r-1}\left(\overrightarrow{k^r}-\overrightarrow{\Delta^r}\right)\right)}{\alpha\left(n+1\right)} \\ &\quad *\left(\frac{2}{n+1}\sum_{j=1}^{n}\frac{\partial\Theta_j^{r-1}\left(\overrightarrow{k^r}-\overrightarrow{\Delta^r}\right)}{\partial k_i^r}-\Theta_i'^{r-1}\left(\overrightarrow{k^r}-\overrightarrow{\Delta^r}\right)+\frac{n-1}{n+1}\right) \\ &\quad +\frac{\sum_{j=1}^{n}\frac{\partial\Theta_j'^{r-1}\left(\overrightarrow{k^r}-\overrightarrow{\Delta^r}\right)}{\partial k_i^r}-1}{\alpha\left(n+1\right)}\left(\Delta_i^r+\Theta_i^{r-1}\left(\overrightarrow{k^r}-\overrightarrow{\Delta^r}\right)\right). \qquad (40)\end{aligned}$$

This can be solved in special cases.

B.1 Only one generator i holds transmission contracts

If $k_j=0$ for all $j\neq i$ and generators do not buy negative quantities of contracts, then $\Delta_j^r=0$. We can simplify notation $k=k_j$, $\Delta^r=\Delta_i^r$ so (40) becomes

$$\begin{aligned}0 &= \frac{n-1}{n+1}\frac{Q_{comp}+k^r-\Delta^r-\Theta^{r-1}\left(k^r-\Delta^r\right)}{\alpha\left(n+1\right)}\left(1-\Theta'^{r-1}\left(k^r-\Delta^r\right)\right) \qquad (41)\\ &\quad +\frac{\Theta'^{r-1}\left(k^r-\Delta^r\right)-1}{\alpha\left(n+1\right)}\left(\Delta_i^r+\Theta^{r-1}\left(k^r-\Delta^r\right)\right).\end{aligned}$$

(41) is either satisfied if

$$\Theta'^{r-1}\left(k^r-\Delta^r\right)=1, \qquad (42)$$

or if

$$\Delta^r+\Theta^{r-1}\left(k^r-\Delta^r\right)=\frac{n-1}{2n}\left(Q_{comp}+k^r\right). \qquad (43)$$

In the last period $r=1$ we have $\Theta^{r-1}\left(k^r-\Delta^r\right)=0$ and (43) gives $\Delta^1=\frac{n-1}{2n}\left(Q_{comp}+k^1\right)$, implying

$$\begin{aligned}\Theta^1\left(k^2-\Delta^2\right) &= \Delta^2+\Delta^1=\Delta^2+\frac{n-1}{2n}\left(Q_{comp}+k^2-\Delta^2\right), \qquad (44)\\ &= \frac{n-1}{2n}\left(Q_{comp}+k^2\right)-\frac{n+1}{2n}\Delta^2.\end{aligned}$$

In the period $r=2$ (43) gives:

$$\Delta^2+\Theta^1\left(k^2-\Delta^2\right)=\frac{n-1}{2n}\left(Q_{comp}+k^2\right). \qquad (45)$$

Substituting (44) in gives

$$\Delta^2=\frac{n+1}{2n}\Delta^2,$$

and therefore $\Delta^2=0$ for $n>1$. Applying similar arguments $\Delta^r=0$ for $r>2$. The single holder of transmission contracts will only sell rights in the last period. This corresponds to the monopolist in Allaz and Vila (1993) not falling into the forward selling trap.

B.2 Assume all generators hold transmission contracts

Solving (40) for the last retrading period $r = 1$ we know that subsequently no further rights can be resold $\Theta_{j,i}^{r-1}\left(\overrightarrow{k^r} - \overrightarrow{\Delta^r}\right) = 0$. This implies that

$$\Delta_i^1 = \left(Q_{comp} + \sum_{j=1}^{n}\left(k_j^1 - \Delta_j^1\right)\right)\frac{n-1}{n+1}. \tag{46}$$

Summing (46) over all i and resubstituting the expression for $\sum_{j=1}^{n} \Delta_i^1$ gives

$$\Theta_i^1\left(\overrightarrow{k^1}\right) = \Delta_i^1 = \left(Q_{comp} + \sum_{j=1}^{n} k_j^1\right)\frac{n-1}{n^2+1}. \tag{47}$$

A similar argument shows that [8]

$$\Theta_i^2\left(\overrightarrow{k^2}\right) = n\frac{n-1}{1-n^2+n^3+n}\left(Q_{comp} + \sum_{j=1}^{n} k_j^2\right). \tag{52}$$

(47) and (52) show that we can write

$$\Theta_i^r\left(\overrightarrow{k^r}\right) = \theta^r\left(Q_{comp} + \sum_{j=1}^{n} k_j^r\right),$$

where $0 \le \theta^r \le 1$, and that generators will sell the same amount of transmission contracts $\Delta_i^r = \Delta_j^r$. This simplification depends on equal and constant marginal costs. Further substitutions give

[8] Using (47) in (40) for $r = 2$ gives and $\overrightarrow{k^1} = \overrightarrow{k^2} - \overrightarrow{\Delta^2}$ gives:

$$\Delta_i^2 + \Theta_i^1\left(\overrightarrow{k^2} - \overrightarrow{\Delta^2}\right) = \left(Q_{comp} + \sum_{j=1}^{n}\left(k_j^2 - \Delta_j^2 - \Theta_j^1\left(\overrightarrow{k^2} - \overrightarrow{\Delta^2}\right)\right)\right)n\frac{n-1}{n+1}, \tag{48}$$

and therefore

$$\Delta_i^2 = \frac{(n-1)^2}{1+n^2}\left(Q_{comp} + \sum_{j=1}^{n}\left(k_j^2 - \Delta_j^2\right)\right). \tag{49}$$

The number of transmission rights sold Δ_i^2 is again symmetric independent of k_j (the lhs is the same for all i):

$$\Delta_i^2 = \frac{(n-1)^2}{1-n^2+n^3+n}\left(Q_{comp} + \sum_{j=1}^{n} k_j^2\right), \tag{50}$$

Therefore

$$\begin{aligned}\Theta_i^2\left(\overrightarrow{k^2}\right) &= \Delta_i^1\left(\overrightarrow{k^2} - \Delta_i^2\right) + \Delta_i^2\left(\overrightarrow{k^2}\right) \\ &= n\frac{n-1}{1-n^2+n^3+n}\left(Q_{comp} + \sum_{j=1}^{n} k_j^2\right).\end{aligned} \tag{51}$$

a recursive relation for θ^r:[9]

$$\theta^r = \theta^{r-1} + \left(1 - n\theta^{r-1}\right) \frac{n - 1 - 2\theta^{r-1}}{n^2 - 2n\theta^{r-1} + 1}. \tag{54}$$

θ^r is increasing in r towards the limit of $\lim_{r\to\infty} \theta^r = 1/n$. Assuming that netting is possible, and generators can buy export contracts, then in the limit generators in aggregate have sold all market power enhancing export contracts and replace these by contracts to mitigate market power such that they have all their output contracted forward. If netting is not allowed generators will substitute the remaining fraction of transmission contracts they would like to buy by financial hedging contracts signed with a third party. If third parties are not available to sign such contracts, then θ^r is limited to the amount of hedging contracts available:

$$\theta^r \leq \frac{1}{n} \frac{K + \sum_{j=1}^{n} k_j}{Q_{comp} + \sum_{j=1}^{n} k^r} \quad \forall r = 1..R.$$

C Continuous retrading

Proposition 10 *It is profitable to sell contracts if $n - m$ people sold before and all others will follow for $m \leq \sqrt{\frac{Q_{comp}}{k}} \left(\sqrt{(n+1) + \frac{Q_{comp}}{k}} - \sqrt{\frac{Q_{comp}}{k}} \right)$ while it is not profitable to sell contracts if every one else follows for $m > \sqrt{\frac{Q_{comp}}{k}} \left(\sqrt{(n+1) + \frac{Q_{comp}}{k}} - \sqrt{\frac{Q_{comp}}{k}} \right)$.*

Proof. Let $\Delta\pi^s_{l,j}$ be the profit for a generator from selling all k transmission contracts if l generators sold before and j generators, including the one assessed, will subsequently sell. Let $\pi_{s,y}$ be profits of a generator if he sells (s=1) or retains (s=0) his transmission contracts and a total of y generators sell.

$$\begin{aligned} \Delta\pi^s_{l,j} &= \pi_{1,l+j} - \pi_{0,l} \\ &= (p_{l+j} - c_2)\, q_{s=1,l+j} + (p_{l+j} - c_1)\, k \\ &\quad - ((p_l - c_2)\, q_{s=0,l} + (p_l - c_1)\, k) \end{aligned}$$

[9] Substituting in (40) gives:

$$\Delta_i^r = \frac{n - 1 - 2\theta^{r-1}}{n^2 + 1 - 2n\theta^{r-1}} \left(Q_{comp} + \sum_{j=1}^{n} k_j^r \right). \tag{53}$$

Using the definition of of Θ^r:

$$\Theta^r(k^r) = \Delta^r + \Theta^{r-1} \left(\sum_{j=1}^{n} k_j^r - n\Delta_i^r \right),$$

or θ^r respectively and substitute Δ_i^r from (53) we obtain (54).

Substituting p and q[10] from (39) gives[11]

$$\begin{aligned}\Delta\pi^s_{l,j} &= \pi_{s,l+j} - \pi_{0,l} \\ &= \frac{k}{\alpha(n+1)^2}\left(\left(n+1-2j\right)Q_{comp} + \left(\left(n+1-2j\right)\left(n-l\right) + \left(j-n-1\right)j\right)k\right) \end{aligned} \quad (55)$$

We want to know the max m for which $\Delta\pi^s_{n-m,n} > 0$. Using (55) and setting $l = n - m$ and $j = m$ gives:

$$(n+1-2m)\,Q_{comp} - m^2 k > 0,$$

which is satisfied for

$$m \leq \sqrt{\frac{Q_{comp}}{k}}\left(\sqrt{(n+1) + \frac{Q_{comp}}{k}} - \sqrt{\frac{Q_{comp}}{k}}\right). \quad (56)$$

■

If we assume that import capacity K does not exceed competitive local generation capacity Q then the minimum value for $\frac{Q_{comp}}{k}$ is n and therefore for $n = 3,4,5,6,7,8$ generators $m + 1 = 2,3,3,3,4,4$ generators will retain their transmission contracts. If we assume instead that transmission contracts are only a small fraction of total output, $\frac{Q_{comp}}{k} \to \infty$ then (56) turns to $m < \frac{n+1}{2}$ and therefore a slightly larger fraction of generators $m + 1 = 2,3,3,4,4,5$ will retain transmission contracts.

Proposition 11 *Assume $m+1+h$ generators hold contracts, then it is profitable for one of these generators to sell his contracts, if subsequently $m+1$ generators retain contracts as long as* $h < \sqrt{\frac{Q_{comp}}{k} + (m+1)}\left(\sqrt{\frac{Q_{comp}}{k} + m + n + 2} - \sqrt{\frac{Q_{comp}}{k} + (m+1)}\right)$.

Proof. Setting $l = n - m - 1 - h$ and $j = h$ in (55) gives $\Delta\pi^s_{l,j} > 0$ for

$$h < \sqrt{\frac{Q_{comp}}{k} + (m+1)}\left(\sqrt{\frac{Q_{comp}}{k} + m + n + 2} - \sqrt{\frac{Q_{comp}}{k} + (m+1)}\right) \quad (57)$$

10

$$\begin{aligned} p_y &= \frac{Q_{comp} + (n-y)\,k}{\alpha(n+1)} + c_2 \\ q_{s,y} &= \frac{Q_{comp} + (n-y)\,k}{n+1} - (1-s)\,k \end{aligned}$$

11

$$\begin{aligned} = \ & \frac{1}{\alpha}\left(\frac{A - K - \alpha c_2 + (n-l)\,k}{n+1} - \frac{jk}{n+1}\right)\left(\frac{A - K - \alpha c_2 + (n-l)\,k}{n+1} - \frac{jk}{n+1}\right) \\ & + \left(\frac{-jk}{\alpha(n+1)}\right)k \\ & - \frac{1}{\alpha}\left(\frac{A - K - \alpha c_2 + (n-l)\,k}{n+1}\right)\left(\frac{A - K - \alpha c_2 + (n-l)\,k}{n+1} - k\right) \end{aligned}$$

■

Calculating again the lower bound based on the previously calculated m and assuming that $k = \frac{Q_{comp}}{n}$ we obtain for $n = 3, 4, 5, 6, 7, 8$ that $h = 1, 2, 2, 3, 3, 3$.

For $n \leq 7$ we obtain the result that the first generators will always sell their transmission contracts because $m + h + 1 \geq n$. For $n = 8$ the $m + h + 1 = 7$ and therefore it is not profitable for the first generator to sell his transmission contracts. As any one generator makes the same calculation no generator will sell his transmission contracts. The development is slightly unregular due to the integer effects, such that for $n = 9$ the first generator again sells his contracts, whereas for $10 \leq n \leq 14$ again no generator sells. For $n > 14$ we obtain that $m + h + 2 < n$ and therefore one could expect that the the first generators sell, until $m + h + 2$ generators are left with contracts.

Proposition 12 *If additional sales are always possible and netting is inhibited, then less than half of symmetric generators will sell their market power enhancing transmission contracts.*

D Pay-as-bid auction equilibrium

Joskow and Tirole (2000) analysed a 'pay-as-bid' auction for a monopoly generator buying transmission contracts and we replicate their equations (1) and (2) in equations (58) and (62). We first consider the mixed strategy equilibrium for the monopoly case and derive the equilibrium with n Nash-Cournot oligopolists in Appendix E.

As before, we assume that traders perfectly arbitrage any profit opportunity. Let $k_m(\eta)$ be the monopolist's bid schedule. Following Joskow and Tirole, the mixed strategy Nash equilibrium which satisfies the condition that traders make on expectation non-negative profits is defined by two functions. The first is the distribution function $H(.)$ from which the monopolist draws his bid schedule $\eta \,\epsilon\, (\underline{\eta}, \overline{\eta})$ with $Prob(\eta \leq x) = H(x)$. The second is the aggregate bid function of traders, $k_t(\eta)$, where $k_t(\eta) + k_m(\eta) = K$. The generator secures $max(0, K - k_t(\eta))$. The distribution function $H(.)$ has to ensure that traders make zero expected profit and the equilibrium bid schedule of traders has to satisfy the condition that the monopolist is indifferent between choosing any η from the support $(\underline{\eta}, \overline{\eta})$ of $H(.)$.

D.1 Equilibrium bid schedule of traders

The profit of the monopolist consists of profit in the energy market minus the cost of buying transmission contracts. Because the monopolist's capacity purchase is always the difference between total capacity and the traders' bid schedule, we have:

$$\pi_m^{auction}(K - k_t(\eta)) = \frac{(Q_{\text{comp}} + K - k_t(\eta))^2}{4\alpha} + (K - k_t(\eta))(c_2 - c_1 - \eta). \tag{58}$$

For the monopolist to be indifferent between any η, his profit must be constant as η changes. Setting the first derivative to zero gives

$$\frac{\partial k_t(\eta)}{\partial \eta} = -\left[\frac{K - k_t(\eta)}{\frac{Q_{\text{comp}}+2\alpha(c_2-c_1)}{2\alpha} + \frac{k_m(\eta)}{2\alpha} - \eta}\right] = -\left[\frac{K - k_t(\eta)}{\eta_0 + \frac{1}{2\alpha}k_m - \eta}\right], \tag{59}$$

where η_0 is the value of transmission contracts for traders when the monopolist does not own contracts. The solution of the differential equation (59) is

$$k_t(\eta) = K - 2\alpha\left(\eta - \eta_0 + \sqrt{(\eta - \eta_0)^2 - \frac{K}{2}const_1}\right). \tag{60}$$

D.2 Equilibrium monopoly bid distribution function

The distribution $H(\cdot)$ is such that traders make zero profit if and only if their aggregate bid schedule is the one calculated in (60). The expected value of a marginal bid equals the integral over all bids by the monopolist that are lower than the bid price η of the trader, weighted with their probability:

$$E\left[v_t(\eta)\right] = \int_{\underline{\eta}}^{\eta} H\prime(\widetilde{\eta})\left(p_2(\widetilde{\eta}) - p_1 - \eta\right) d\widetilde{\eta}.$$

As profit has to be zero for all η, the change in profit also has to be zero:

$$\frac{\partial}{\partial \eta}E\left[v_t(\eta)\right] = H\prime(\eta)\left(p_2(\eta) - p_1 - \eta\right) - H(\eta) \equiv 0. \tag{61}$$

The upper support of the bids is $\overline{\eta}$ with $H(\overline{\eta}) = 1$. Solving (61) gives

$$H(\eta) = \exp\left[-\int_{\eta}^{\overline{\eta}} \frac{1}{p_2(\widetilde{\eta}) - p_1 - \widetilde{\eta}} d\widetilde{\eta}\right]. \tag{62}$$

Substituting linear demand p_2 from (36), assuming Cournot competition ($\tau = 0$), and $k_m(\eta) = K - k_t(\eta)$ from (60) into (62) we obtain

$$H(\eta) = \exp\left[-\int_{\eta}^{\overline{\eta}} \frac{1}{\sqrt{(\widetilde{\eta} - \eta_0)^2 - \frac{K}{2}const_1}} d\widetilde{\eta}\right] = \frac{\eta - \eta_0 + \sqrt{(\eta - \eta_0)^2 - \frac{K}{2}const_1}}{\overline{\eta} - \eta_0 + \sqrt{(\overline{\eta} - \eta_0)^2 - \frac{K}{2}const_1}}. \tag{63}$$

At the lower end of the support $\underline{\eta}$, $H(\underline{\eta}) = 0$. This gives us $\underline{\eta} - \eta_0 + \sqrt{(\underline{\eta} - \eta_0)^2 - \frac{K}{2}const_1} = 0$, which is only satisfied for $\underline{\eta} = \eta_0$ and $const_1 = 0$. We therefore obtain:

$$k(\eta) = 4\alpha(\eta - \eta_0), \quad \overline{\eta} = \eta_0 + \frac{K}{4\alpha}, \quad H\prime(\eta) = \frac{4\alpha}{K}. \tag{64}$$

E Mixed strategy Equilibrium in Oligopoly Case

An oligopoly with $n > 1$ generators is located at the importing node. In a "pay-as-bid" auction any generator will either not participate in the auction or buy all transmission contracts available. Bids are chosen from a continuous distribution, therefore no more than one generator obtains transmission contracts.

E.1 Energy market

To simplify our subsequent calculations we define η as the margin paid above the value transmission contracts take if generators own no contracts. We know from (39) that if a generator owns k contracts then the value of contracts is increased due to the decrease in output:

$$\Delta p_{2,k} = \frac{k}{\alpha (n+1)}. \tag{65}$$

With retrading generators sell subsequently $\Delta = \min(\frac{n-1}{2n}(Q_{comp} + k), k)$ of their transmission contracts. As we will later confirm $\Delta < k$, so we can write the price increase as a function of the number of transmission contracts bought in the auction:

$$\Delta p_{2,k,r} = \frac{k}{2n\alpha} - \frac{n-1}{2n\alpha (n+1)} Q_{comp}. \tag{66}$$

The change in profits for a generator buying k transmission contracts is, using (39):

$$\begin{aligned} \Delta\pi_k^{change} &= q_{i,k}(p_{2,k} - c_2) - q_{i,0}(p_{2,0} - c_2) + \Delta p_{2,k} k \\ &= \frac{k}{\alpha (n+1)^2}(k - (n-1) Q_{comp}), \end{aligned} \tag{67}$$

and if the generator resells contracts the expression is:

$$\Delta\pi_{k,r}^{change} = \frac{1}{4\alpha n (n+1)^2}((n+1)k - (n-1)Q)^2. \tag{68}$$

If no retrading is feasible, then generator only profitably obtain contracts at the price contracts have if no generator owns contracts, if $k > (n-1) Q_{comp}$ (67). If the generator can sell some of these contracts at the ex-post price, then it is profitable for the generator even if he obtains fewer contracts $k > \frac{n-1}{n+1} Q_{comp}$ at the price which contracts take if no generator holds such contracts.(68). The generator can resell some of the contracts he obtained at higher prices, assuming that traders trust or market design ensures that he will not resell additional contracts in subsequent periods.

Similarly the change in profits $\Delta\pi_{-k}^{change}$ is calculated for a generator assuming a competing generator obtains contracts:

$$\begin{aligned} \Delta\pi_{-k}^{change} &= q_{i,-k}(p_{2,k} - c_2) - q_{i,0}(p_{2,0} - c_2) \\ &= \frac{1}{\alpha} \frac{k}{(n+1)^2}(k + 2Q_{\text{comp}}), \end{aligned} \tag{69}$$

and if the competing generator resells some of his contracts:

$$\Delta\pi_{-k,r}^{change} = \frac{((n+1)k - (n-1)Q)}{4\alpha n (n+1)^2}((n+1)k + (3n+1)Q). \tag{70}$$

E.2 Game structure

The game with an oligopoly of generators is the same as with a monopolist. Generators and traders submit their bid schedule and the auctioneer accepts the highest bids. Generators either submit no bid or bid for all transmission contracts available at price η. Generators draw the bid price η from the interval $[\underline{\eta}, \overline{\eta}]$ with probability $Prob(\eta < x) = H(x)$. As the interval is continuous the probability that more than one generator submits a bid of the same price is zero.

E.3 Distribution from which generators draw their bids

We now calculate the aggregate density function $H(\eta)$ describing the probability $H'(\eta)$ with which a generators chooses a bid η. The generator decides on the distribution function $H(\eta)$ such that he can ensure that all traders make zero profits if and only if the aggregate bid schedule of traders is $1 - k(\eta)$.

Generators can only profitably bid if they obtain a significant positive quantity of transmission contracts and hence $H(0) > 0$ and $H(\eta) = H(0)$ for $\eta \epsilon [0, \underline{\eta}]$.

The expected value $v_t(\eta)$ of marginal bid by a trader is zero if the bid is not accepted. If the bid is accepted, then $v_t(\eta)$ is the weighted integral over price increase minus bid price:

$$E[v_t(\eta)] = \int_{\underline{\eta}}^{\eta} nH(\widetilde{\eta})^{n-1} H'(\widetilde{\eta}) \Delta p_{2,k} d\widetilde{\eta} - H(\eta)^n \eta \qquad \eta \epsilon (\underline{\eta}, \overline{\eta}). \tag{71}$$

After substituting $\Delta p_{2,k}$ from (65) (or (66) for the case with retrading) we differentiate the right hand side with respect to η. The left hand side is zero, because profits for all trades are zero and therefore also the differential is zero. Rearranging we obtain for the case without and with retrading:

$$\frac{H'(\eta)}{H(\eta)} = \frac{1}{n} \frac{\alpha(n+1)}{k - \eta\alpha(n+1)} \qquad \frac{H_r'(\eta)}{H_r(\eta)} = \frac{2\alpha}{k - \frac{n-1}{(n+1)} Q_{comp} - 2n\alpha\eta}. \tag{72}$$

Setting $\eta = \underline{\eta}$ in (71) gives $E\left[v_t\left(\underline{\eta}\right)\right] = -H(\underline{\eta})^n \underline{\eta}$. Traders submitting bids at $\underline{\eta} > 0$ would make a loss, therefore we conclude that $\underline{\eta} = 0$.

E.4 Traders' aggregate bid schedule

We now calculate which distribution of bid function $1 - k(\eta)$ of traders ensures that generators will be indifferent between not bidding or bidding any value on the interval $[\underline{\eta}, \overline{\eta}]$. Generator's expected profits when submitting a bid (η, K) consist of the profits of winning, given by the probability that other generators bid lower $H(\eta)^{n-1}$ times profits minus costs for transmission contracts. Furthermore we add the weighted profits made when other generators obtain transmission contracts and therefore reduce their output and push up prices. All these profits have to equal the profits from not participating in the auction:

$$H(\eta)^{n-1}\left[\Delta\pi_{k(\eta)} - \eta k(\eta)\right] + \int_{\eta}^{\overline{\eta}} (n-1)\, H(\widetilde{\eta})^{n-2} H'(\widetilde{\eta}) \Delta\pi_{-k(\widetilde{\eta})} d\widetilde{\eta}$$
$$= \int_{\underline{\eta}}^{\overline{\eta}} (n-1)\, H(\widetilde{\eta})^{n-2} H'(\widetilde{\eta}) \Delta\pi_{-k(\widetilde{\eta})} d\widetilde{\eta}. \tag{73}$$

Differentiation (73) with respect to η and then substituting $H'()$ from (72) and $\Delta\pi$'s from (67) and (70) provided a differential equation to describe $k(\eta)$:

$$\frac{\partial k}{\partial \eta} = \alpha (n+1)^2 k \frac{1 + \frac{n-1}{n} \frac{Q_{\text{comp}} + \alpha(n+1)\eta}{k - \eta\alpha(n+1)}}{2k - \alpha(n+1)^2 \eta + (1-n) Q_{\text{comp}}}. \tag{74}$$

For the case with retrading, substitute (68) and (70) in (73) and differentiate with respect to η to obtain:

$$\frac{\partial k_r}{\partial \eta} = 2\alpha n \frac{\left(2\frac{n-1}{n+1} Q_{comp} + k\right)\left(k - \frac{n-1}{(n+1)} Q_{comp}\right) - 2\alpha\eta k}{\left(k - \frac{n-1}{(n+1)} Q_{comp} - 2n\alpha\eta\right)^2}. \tag{75}$$

E.5 Boundary conditions

We require that at the upper end of the density function $H(\overline{\eta}) = 1$ a generator buys all transmission rights $k(\overline{\eta}) = K$. If $k(\overline{\eta}) > K$ then generators could already obtain all transmission contracts for a lower η' and therefore they will not submit any bids above η' and therefore $H(\eta') = 1$ for $\eta' < \overline{\eta}$. If on the other hand $k(\overline{\eta}) < K$, then a generator increasing his bid to $\overline{\eta} + \varepsilon$ would obtain all transmission contracts. This would be a profitable deviation.

From (71) we know that $\underline{\eta} = 0$ and to ensure that generators are indifferent between bidding and not bidding we require $\Delta\pi_{k(\underline{\eta})}^{change} = 0$, which defines $k(\underline{\eta})$ as $k(\underline{\eta}) = \frac{n-1}{n+1} Q_{comp}$ in the case of no retrading and $k_r(\underline{\eta}) = \frac{1}{n+1} Q_{comp}$ in the case of retrading.

E.6 Numerical Example

A numerical example demonstrates the existence of a mixed strategy equilibrium. With parameter values $n = 2$, $\alpha = 1$, $K = 3$ and $Q_{comp} = 3/2$ the differential equations (72) and (74) are:

$$\frac{\partial k(\eta)}{\partial \eta} = \frac{9}{2} k \frac{4k - 6\eta + 3}{(k - 3\eta)(4k - 18\eta - 3)}, \quad \frac{\partial H(\eta)}{\partial \eta} = \frac{1}{2} \frac{H(\eta)}{\frac{1}{3}k - \eta},$$

and in the case with retrading (72) and (75) are

$$\frac{\partial k_r(\eta)}{\partial \eta} = \frac{16k^2 + 4k - 40\eta k - 6}{(1 + 8\eta - 2k)^2}, \quad \frac{\partial H_r(\eta)}{\partial \eta} = \frac{4H(\eta)}{2k - 1 - 8\eta}.$$ [12]

This can be solved numerically and results for one example are presented in Figure 5.

[12] The denominator of $\frac{\partial H_r(\eta)}{\partial \eta}$ approaches zero for $k- > \frac{1}{2}$, making it necessary to perform the numerical integration starting from positive η.

E.7 Stability

We calculated a Nash equilibrium, which requires stability given other players' bid functions. To check for stability or existence of this equilibrium it should not be profitable for a generator to announce not to participate in the auction. An announced deviation could be profitable even if the previous analysis determined a Nash equilibrium, because other players change their bid functions following the announcement.

The intuitive argument why such a deviation is not profitable goes as follows. To a first order, all generators are indifferent between bidding and not bidding, therefore the remaining generators increase their bidding probability by $1/n$. Traders do not observe any difference because the aggregate distribution of bids stays the same and so they maintain their bidding strategy.

Generators, however, observe a second order effect. If they do not bid, then the probability that someone else will bid has been reduced by $1/n^2$ and therefore the expected profit has been reduced. If they bid a low price, then the probability that someone else bids a higher price has likewise been reduced by $1/n^2$, once again resulting in a reduction of expected revenue, but by less than $1/n^2$. Only if generators bid the maximum price could profit stay the same as before. Therefore generators will submit high bids more frequently. As a result the profits for traders increase. However, we assume perfect arbitrage, therefore additional traders will submit bids. These additional bids reduce the amount of transmission contracts generators can obtain for a given price and therefore the profits of generators. As a result the high bids become less profitable for generators. So generators reaction to the second order effect pushes up revenue for low bids while at the same time traders' reaction reduces revenue for the high bids until generators are indifferent between no, low and high bids. Overall profits for generators are reduced, but by not more than $1/n^2$. This shows that the deviation strategy is not profitable for generators.

F Uniform price auction in meshed networks

We first give the direct proof for a symmetric generators with constant marginal costs (Lemma 13) and then the extended version for asymmetric players with increasing marginal costs (Lemma 14 and following).

Lemma 13 *If constrained transmission capacity in a meshed network is sold on a single price auction that is efficiently arbitraged by traders who can accurately predict future equilibrium spot prices, then symmetric oligopolists with constant marginal costs will only acquire contracts that mitigate market power.*

Proof. Assume that the set of transmission constraints that is binding does not change with the allocation of transmission contracts and that market power is only exercised at one node. As all transmission constraints stay binding and demand is linear, prices are some linear function of demand. Assume the oligopoly generators are located at node 2 with constant marginal costs c_2. Oligoplist i produces q_i, and total production at node 2 is Q. Oligopolists sell to node 3, where the price is p_3 (which can be several nodes, in which case p_3 is a linear combination of prices of several nodes). Prices at node 3 depend on constant competitive production costs, subsumed into A_3, and on output at node 2, where α_3 can be positive, zero or negative:

$$p_2 = A_2 - \alpha_2 Q; \qquad p_3 = A_3 - \alpha_3 Q. \tag{76}$$

In the energy market the oligopolist owning k_i transmission contracts maximises the profit function:

$$\pi_i^{spot} = (p_2 - c_2)\, q_i + (p_3 - p_2)\, k_i \theta, \tag{77}$$

where θ is the ratio of the amount that can be sold to node 3 to the underlying flow-gate rights ($\theta = 3$ in (33)). The FOC gives the optimal output choices in the energy market:

$$q_i = \frac{A_2 - c_2 - (\alpha_2 - \alpha_3) \sum k_i \theta}{\alpha_2 (n+1)} + \frac{\alpha_2 - \alpha_3}{\alpha_2} k_i \theta; \qquad Q = \frac{n (A_2 - c_2) + (\alpha_2 - \alpha_3) \sum k_i \theta}{(n+1)\, \alpha_2}. \tag{78}$$

This allows the determination of energy market profits as a function of transmission contracts. In a uniform price auction competitive arbitrageurs will ensure that the market clearing price for transmission contracts will equal the value of these rights in the energy auction $(p_3 - p_2)$. A generator chooses the number of transmission contracts he bids for in the auction to maximise profits in the energy auction π_{energy} minus the costs he incurs to buy the rights, $(p_3 - p_2)\, k_i \theta$.

$$\begin{aligned} \pi_i^{auction} &= \pi^{spot}(k_i) - (p_3 - p_2)\, k_i \theta \\ &= \frac{A_2 - c_2 - (\alpha_2 - \alpha_3) \sum k_i \theta}{(n+1)} \left(\frac{A_2 - c_2 - (\alpha_2 - \alpha_3) \sum k_i \theta}{\alpha_2 (n+1)} + \frac{\alpha_2 - \alpha_3}{\alpha_2} k_i \theta \right). \end{aligned}$$

Calculating the first order condition with respect k_i, using symmetry between all generators and resubstituting in 78 gives:[13]

$$\sum k_i \theta = \frac{n^2 - n}{1 + n^2} \frac{A_2 - c_2}{\alpha_2 - \alpha_3}; \qquad Q_{transmission\_rights} = \frac{n^2}{1 + n^2} \frac{A_2 - c_2}{\alpha_2}.$$

If output reductions of generators increase prices at the original node more than at the destination node of the transmission contract ($\alpha_2 > \alpha_3$), then generators will buy a positive quantity of transmission contracts (corresponding to an export contract), otherwise a negative quantity.

[13] We assume that less transmission contracts than available capacity are obtained by generators ($|\sum \lambda_i| < K$). Otherwise the argumentation parallel to section (5.1) shows that generators pre-empt all available transmission capacity mitigating market power.

The result can be compared with the output choice $Q_{comp} = \frac{A_2 - c_2}{\alpha_2}$ in a competitive scenario $(n \to \infty)$, and with the output Q_{no_tr} in a situation without transmission contracts ($k_i = 0$ $\forall i$).

$$Q_{no\_tr} = \frac{n}{n+1} Q_{comp} \ < \ Q_{transmission\_rights} = \frac{n^2}{1+n^2} Q_{comp} \ < \ Q_{comp}.$$

As all nodes but node two are competitive, the only deadweight loss will occur due to withholding at node two. Deadweight losses are reduced if generators' market power is mitigated and they withhold less output, as demonstrated. Therefore transmission contracts allocated through a uniform auction are efficiency improving. ■

To generalise the previous lemma to allow for asymmetric generators and increasing marginal costs we require an indirect proof. Lemma 15 calculates output choice in the energy market. Aggregate output is an increasing function of the weighted sum of transmission contract holding. We therefore have to show that this weighed sum is positive to prove that market power is mitigated. Lemma 16 gives the FOC for the transmission contracts auction. Lemma 17 shows that the LHS of the FOC is positive and Lemma 18 proves that the weighted sum of transmission contracts holdings is positive given conditions on the slope the marginal cost curve. These conditions should be easily satisfied, because electricity demand is very inelastic (demand slope high) while marginal costs of generators that can alter their output decision are comparatively flat.

Lemma 14 *If constrained transmission capacity in a meshed network is sold in a single price auction that is efficiently arbitraged by traders who can accurately predict future equilibrium spot prices, then oligopolists will only acquire contracts that mitigate market power. Marginal costs of generation should not increase by more than* $\left(\sqrt{2} - 1\right)$ *times demand slope. If generators are asymmetric, then marginal costs of each generator should not increase by more than* $1/n$ *times demand slope (lower bounds due to approximations).*

Proof. We maintain the demand structure (76) but change the production cost incurred by generators from $C(q_i) = cq_i$ to $C(q_i) = c_i q_i - \frac{\beta_i}{2} q_i^2$. This allows the representation of the local shape of any cost curve.

The proof strategy is to show that the LHS of equation (80), which follows below, is positive if generators are symmetric (because $\bar{c} = c_i$) or if generators are asymmetric and $\forall i$ $\beta_i \leq \alpha_2/n$ (Lemma 17). From Lemma (16) follows that the RHS of (80) is positive and using Lemma (80) and $\forall i$ $\beta_i \leq \sqrt{2} - 1$ it follows that $(\alpha_2 - \alpha_3) \sum \frac{k_i}{\alpha_2 + \beta_i}$ is positive. Finally it follows from Lemma (15) that aggregate output is increased relative to a scenario without transmission contracts and therefore market power mitigated. ■

Lemma 15 *Total output of generators is increasing with* $(\alpha_2 - \alpha_3)\, \theta \sum \frac{k_i}{\alpha_2 + \beta_i}$.

Proof. In the energy market the oligopolist owning k_i transmission contracts maximises the profit function

$$\pi_i^{spot} = \left(A_2 - \alpha_2 Q - c_i - \frac{\beta_i}{2} q_i\right) q_i + (A_3 - A_2 + (\alpha_2 - \alpha_3) Q) k_i \theta.$$

The FOC gives the optimal output choice for a given allocation of transmission contracts k_i:

$$Q = \frac{A_2 \sum \frac{1}{\alpha_2 + \beta_i} - \sum \frac{c_i}{\alpha_2 + \beta_i} + (\alpha_2 - \alpha_3) \theta \sum \frac{k_i}{\alpha_2 + \beta_i}}{1 + \sum \frac{\alpha_2}{\alpha_2 + \beta_i}}, \tag{79}$$

$$q_i = \frac{A_2 - \alpha_2 Q - c_i + (\alpha_2 - \alpha_3) k_i \theta}{\beta_i + \alpha_2}.$$

Iff

$$(\alpha_2 - \alpha_3) \theta \sum \frac{k_i}{\alpha_2 + \beta_i} > 0$$

then aggregate output Q is increased according to (79) and therefore market power mitigated. ■

Lemma 16 *The FOC in the transmission contracts auction, defining* $A = \sum \frac{\alpha_2}{\alpha_2 + \beta_i}$ *and* $\overline{c} = \sum \frac{c_i}{n}$, *is*

$$\frac{A}{1 + nA} (n-1)(A_2 - \overline{c}) - \sum_i \frac{\alpha_2}{\alpha_2 + \beta_i} (\overline{c} - c_i)$$
$$= (\alpha_2 - \alpha_3) \theta \left(\sum \frac{(1+A)^2}{1 + nA} \frac{\beta_i}{\alpha_2} k_i + \sum \frac{\alpha_2}{\alpha_2 + \beta_i} k_i \right). \tag{80}$$

Proof. If transmission contracts are allocated in a uniform price auction, then prices are arbitraged and ownership of transmission contracts only influences the output decision $q_i(k_i)$ of generators but does not directly influence the total profit function

$$\pi_i^{total} = \left(A_2 - \alpha_2 Q - c_i - \frac{\beta_i}{2} q_i\right) q_i. \tag{81}$$

Substituting (79) in (81) and calculating the first order condition with respect to k_i for optimal transmission ownership gives:

$$\left(A_2 + \sum_j \frac{\alpha_2 (c_j - c_i)}{\alpha_2 + \beta_j} - (\alpha_2 - \alpha_3) \theta \sum_j \frac{\alpha_2 k_j}{\alpha_2 + \beta_j} - c_i\right)$$
$$\left(\frac{\alpha_2}{2\alpha_2 + \beta_i} - \frac{\frac{\alpha_2}{\alpha_2 + \beta_i}}{1 + \sum_l \frac{\alpha_2}{\alpha_2 + \beta_l}}\right)$$
$$= \left(\beta_i \sum_j \frac{\alpha_2}{\alpha_2 + \beta_j} + \beta_i + \alpha_2 \frac{\alpha_2}{\alpha_2 + \beta_i}\right) \frac{\alpha_2 - \alpha_3}{2\alpha_2 + \beta_i} k_i \theta. \tag{82}$$

Multiplying (82) by $(2\alpha_2 + \beta_i) \frac{1+A}{1+nA}$ and summing over i and defining $A = \sum \frac{\alpha_2}{\alpha_2 + \beta_i}$ and $\overline{c} = \sum \frac{c_i}{n}$ we obtain (80). ■

Lemma 17 *The LHS of FOC (80) is positive if* $\forall i \quad \frac{\beta_i}{\alpha_2} \leq 1/n$.

Proof. Define $f = \max_i(\frac{\beta_i}{\alpha_2})$ with $f \leq 1/n$. We have to show that the LHS of FOC (80) is positive. The worst case scenario is that m small generators have high marginal costs $c_i = c_0 + c_d$ which are increasing with $\beta_i = f\alpha_2$, while the remaining $n-m$ generators have lower marginal costs $c_i = c_0$ which are constant $\beta_i = 0$. The condition for a positive LHS of FOC (80) can now be written as:

$$\frac{A}{1+nA}(n-1)\left(A_2 - c_0 - \frac{m}{n}c_d\right) - m\frac{n-m}{n}c_d\frac{f}{1+f} > 0.$$

Using $f = 1/n$ and reformating gives

$$A(n-1)(A_2 - c_0 - c_d) + \left(A(n-1) - (1+nA)\frac{m}{n+1}\right)\frac{n-m}{n}c_d > 0. \tag{83}$$

For small generators to participate their marginal costs $c_0 + c_d$ have to be such that demand is positive at their marginal cost, and therefore $A_2 - c_0 - c_d > 0$. We are left to show in (83) that the parentheses of the second term is positive. Using $A = n - m\frac{f}{1+f} = \frac{n+n^2-m}{1+n}$ gives

$$nm^2 - \left(n + 2n^2 + n^3\right)m + \left(n + n^2\right)\left(n^2 - 1\right) > 0.$$

For $m \to \pm\infty$ the inequality is satisfied and the LHS is at the minimum for $m = \frac{n+2n^2+n^3}{2n} > n$. As m has to be smaller than n we are left to show that the LHS is positive for maximum possible $m = (n-1)$. In this case the LHS takes the form $n(n-1)^2 > 0$. ■

Lemma 18 $(\alpha_2 - \alpha_3)\theta \sum \frac{\alpha_2}{\alpha_2+\beta_i}k_i$ *is positive if LHS of FOC (80) is positive and* $\forall_i \quad \frac{\beta_i}{\alpha_2} \leq \sqrt{2} - 1$.

Proof. (by contradiction). Assume that

$$(\alpha_2 - \alpha_3)\theta \sum \frac{\alpha_2}{\alpha_2 + \beta_i}k_i < 0. \tag{84}$$

We know that the LHS of FOC (80) is positive, therefore the RHS is positive:

$$(\alpha_2 - \alpha_3)\theta\left(\sum \frac{(A+1)^2}{1+nA}\frac{\beta_i}{\alpha_2}k_i + \sum \frac{\alpha_2}{\alpha_2 + \beta_i}k_i\right) > 0. \tag{85}$$

Inequality (84) and (85) can only be satisfied simultaneously if for some i with $k_i > 0$ the coefficient of k_i in the first sum exceeds the coefficient in the second sum:

$$\frac{(A+1)^2}{1+nA}\frac{\beta_i}{\alpha_2} > \frac{\alpha_2}{\alpha_2 + \beta_i} \quad \text{or} \quad \frac{\beta_i}{\alpha_2}\frac{\alpha_2 + \beta_i}{\alpha_2} > \frac{1+nA}{(A+1)^2}. \tag{86}$$

Use $f = \max_i(\frac{\beta_i}{\alpha_2})$ to obtain a lower bound for $A = \sum_i \frac{\alpha_2}{\alpha_2+\beta_i} > \frac{n}{1+f}$. Apply the lower bound to the RHS of (86), which is increasing in A, and the upper bound for β_i to the LHS to obtain:

$$f(f+1) > \frac{1 + nn\frac{1}{1+f}}{\left(n\frac{1}{1+f} + 1\right)^2} \quad \text{or} \quad f(n+f+1)^2 > f + 1 + n^2. \tag{87}$$

For our assumption to be true (87) requires that f is bigger than $\begin{array}{cccccc} n & 2 & 3 & 4 & 6 & \infty \\ f & \sqrt{2}-1 & .52 & .57 & .61 & 1 \end{array}$.

If we assume that $f < \sqrt{2}-1$ then inequality (84) and (85) can not be satisfied simultaneously when $(\alpha_2 - \alpha_3)\,\theta \sum \frac{\alpha_2}{\alpha_2+\beta_i} k_i$ is positive. ■

G Mixed-strategy equilibrium in three-node network

The result for the monopoly case is given here. The monopolist will offer to buy all contracts available at price η. He draws η with from the distribution $H()$ with probability $H'(\eta)$. The aggregate bid function of traders does not change in equilibrium because additional bids make losses while fewer bids provide an arbitrage opportunity. The monopolist therefore anticipates the proportion of contracts $k(\eta)$ he can obtain when bidding η in the auction. In Nash equilibrium the monopolist should be indifferent between all prices and therefore indifferent between changing the price of his bid. Subtract the costs for buying transmission contracts ηk from the profits in the energy market (33) to obtain total profits:

$$\Delta\pi_{total}\,(k_i) = \frac{(Q_{\text{comp}} + 3k)^2}{4\alpha\,(n+1)^2} + (c_2 - c_1)\,3k - \eta k. \tag{88}$$

The first order condition (FOC) with respect to the bid price η gives

$$\frac{\partial k(\eta)}{\partial \eta} = \frac{k\,(\eta)}{\frac{3}{8\alpha}Q_{\text{comp}} + 3\,(c_2 - c_1) + \frac{9}{8\alpha}k\,(\eta) - \eta} = \frac{k\,(\eta)}{\frac{\eta_0}{3} + \frac{9}{8\alpha}k\,(\eta) - \eta}. \tag{89}$$

In (89), η_0 is the value of transmission contracts for traders when the monopolist does not own contracts. The solution of the differential equation is

$$k(\eta) = \frac{\eta - \frac{\eta_0}{3} + \sqrt{\left(\eta - \frac{\eta_0}{3}\right)^2 - \frac{9K}{8}const_1}}{\frac{9}{8\alpha}}. \tag{90}$$

The traders' strategy is determined from the no-arbitrage condition that requires all bids to make zero expected profit. A bid is only accepted if the price is at least as high as the monopolist's bid. The expected profit from a bid equals the integral over all bids by the monopolist that are lower than the bid price η of the trader, weighted with their probability:

$$E\left[\pi_t(\eta)\right] = \int_{\underline{\eta}}^{\eta} H\prime\,(\widetilde{\eta})\,(3p_3(\widetilde{\eta}) - 3p_2(\widetilde{\eta}) - \eta)\,d\widetilde{\eta}.$$

As profit has to be zero for all η, the change in profit also has to be zero:

$$\frac{\partial}{\partial \eta}E\left[\pi_t(\eta)\right] = H\prime\,(\eta)\,(3p_3(\eta) - 3p_2(\eta) - \eta) - H\,(\eta) \equiv 0. \tag{91}$$

Differential equation (91) determines the probability with which the monopolist chooses bids for a mixed strategy equilibrium to exist. The upper support of the bids is $\overline{\eta}$ with $H(\overline{\eta}) = 1$. Solving (91) gives

$$H(\eta) = \exp\left[-\int_{\eta}^{\overline{\eta}} \frac{1}{3p_3(\widetilde{\eta}) - 3p_2(\widetilde{\eta}) - \widetilde{\eta}} d\widetilde{\eta}\right]. \tag{92}$$

Substituting p_2 and p_3 from (32) and $k()$ from (90) into (92) we obtain :

$$H(\eta) = \exp\left[-\int_{\eta}^{\overline{\eta}} \frac{1}{\sqrt{\left(\widetilde{\eta} - \frac{\eta_0}{3}\right)^2 - \frac{9K}{8} const_1}} d\widetilde{\eta}\right] = \frac{\eta - \frac{\eta_0}{3} + \sqrt{\left(\eta - \frac{\eta_0}{3}\right)^2 - \frac{9K}{8} const_1}}{\overline{\eta} - \frac{\eta_0}{3} + \sqrt{\left(\overline{\eta} - \frac{\eta_0}{3}\right)^2 - \frac{9K}{8} const_1}}$$

At the lower support $\underline{\eta}$, $H(\underline{\eta}) = 0$. This gives $\underline{\eta} - \frac{\eta_0}{3} + \sqrt{\left(\underline{\eta} - \frac{\eta_0}{3}\right)^2 - \frac{9K}{8} const_1} = 0$, which is only satisfied for $\underline{\eta} = \frac{\eta_0}{3}$ and $const_1 = 0$. We therefore obtain:

$$k(\eta) = \frac{16\alpha}{9}\left(\eta - \frac{\eta_0}{3}\right), \qquad \overline{\eta} = \frac{\eta_0}{3} + \frac{9K}{16\alpha}, \quad H\prime(\eta) = \frac{16\alpha}{9K}. \tag{93}$$

The mixed-strategy equilibrium is thereby described and looks similar to the one presented in figure (4).