



Issues on Market Power Modelling

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Oligopolistic market models uses

- In regulation
 - Qualitative o semi-quantitative results:
 - Absence/Presence of market power.
 - Direction of regulatory measures.
 - Easy to explain
 - Robust
- In management
 - Quantitative results
 - Able to detailed system modelling
 - Prices and physical output forecasting
 - Able to incorporate market data

Models classification (i)

- Supply function equilibria
 - Few questionable data, maybe questionable assumptions (great demand uncertainty) in some market settings.
 - Very difficult to apply if intertemporal constraints are important.
- Agents-based simulation
 - Possibility of simulating very complex system and market settings.
 - Difficulties in modelling agents behaviour: it is a (very) bounded rationality approach.
- Business dynamics
 - Much same advantages and criticisms

Models classification (ii)

- Conjectural variations
 - Flexible approach.
 - It includes Cournot, Bertrand, Stackelberg and other equilibria computing approaches.
 - It allows to model complex systems, although not so complex as it used to be in centralized models.
 - General setting is modelling as Complementary Problems, but then there are serious difficulties to consider integer variables, and solvers are slow and not very reliable for big models.
 - It is also possible to model as Optimisation Problems, but then some models can not be implemented.
 - Difficulties in getting reasonable values of conjectural variations.

A list of issues ...

1. Estimating conjectural variations
2. Inter-temporal constraints
3. Uncertainty
4. Electricity network effects
5. Hydro cascades
6. Integer variables effects
7. Modelling of consecutive markets
8. Interface with other systems (specially gas)
9. ...

Reference model (i)

- Although these issues are relevant for most kind of models, I will focus in a particular family that we have used extensively: **conjectural variations equilibrium computed through optimization.**
- In the single period case (to be generalized later), each utility u **marginal cost** and **marginal revenue** must be equal:

$$MC_u = MR_u$$

Reference model (ii)

- Marginal revenue has two components:
 - 1 additional MW-h earns the market price λ .
 - because of the increased production, market prices diminishes an amount θ_u . This price fall impacts on all the energy which is being traded in the daily market, which is assumed to be generated power

$$MR_u = \lambda - \theta_u P_u$$



Conjectural variation

Reference model (iii)

- There must be also fulfilled:
 - Demand curve

$$D = D^0 - \alpha_0 \lambda$$

- Power balance

$$\sum_u P_u = D$$

Reference model (iv)

- Putting all together

$$MC_u = MR_u$$

$$MR_u = \lambda - \theta_u P_u$$

$$D = D^0 - \alpha_0 \lambda$$

$$\sum_u P_u = D$$

Reference model (v)

- It is easy to check that previous equations are the optimality conditions of problem:

$$\begin{array}{ll} \min_{P_u, D} & \sum_u \left(\overline{C}_u(P_u) \right) - U(D) \\ \text{s.t.} & \sum_u P_u = D \quad : \lambda \end{array}$$

← Price is the multiplier

– Demand utility: $U(D) = \frac{1}{\alpha_0} \left(DD^0 - \frac{D^2}{2} \right)$

– Effective cost: $\overline{C}_u(P_u) = C_u(P_u) + \frac{\theta_u}{2} P_u^2$

Reference model (vi)

- It is possible to include forward contracting (contracts unaffected by market price) in the model:

$$\begin{aligned} \min_{P_u, D} \quad & \sum_u \left(\overline{C}_u(P_u) \right) - U(D) \\ \text{s.t.} \quad & \sum_u P_u = D \quad : \lambda \end{aligned}$$

$$\overline{C}_u(P_u) = C_u(P_u) + \frac{\theta_u}{2} (P_u - F_u)^2$$

Forward contracts

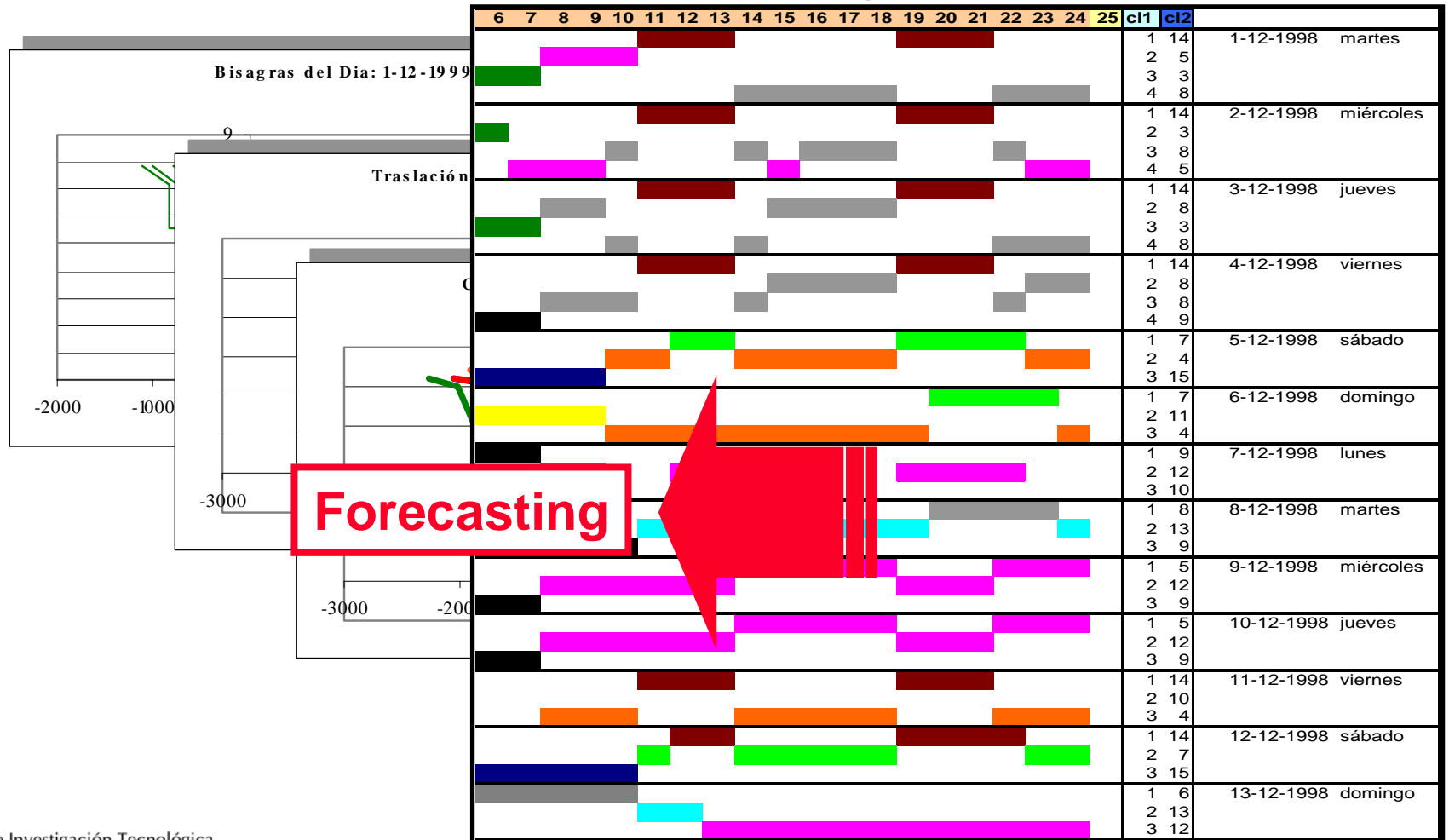


Estimating conjectural variations (i)

- Two main ways:
 1. From bid curves
 2. From market prices (“implied” conjectural variations) and equilibrium models.

Estimating conjectural variations (ii)

- It is possible to try to estimate them from market data, if medium-term modelling is intended.



Estimating conjectural variations (iii)

- If “bid” conjectural variations are used, market simulation can be significantly different of real prices.
 - Why? (unmodelled profit/losses terms?)
 - How to fix (implied conjectural variations? Implied forward contracting?)
- Difficult to carry out to the long-term, or if significant change in electricity market is forecasted (mergers, regulatory changes, ...)
 - Try to estimate from supply function equilibria studies.
 - Try to forecast from Cournot models with some sort of “implied” demand elasticity.

Inter-temporal constraints

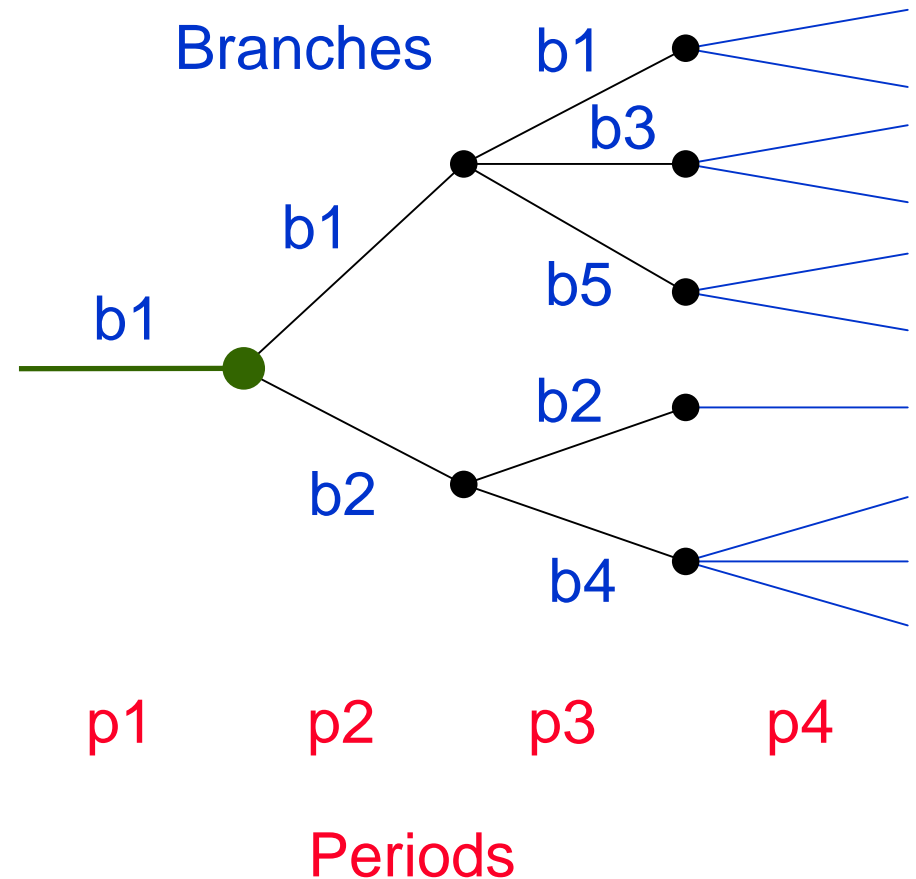
- They can not be ignored for realistic forecasting, especially in systems with significant hydro component.
- But there are also other inter-temporal constraints:
 - Short-term (ramps, shut-down times ...).
 - Medium and long-term (fuel quotas, CTCs payments, ...)
- Difficult to implement in some Complementary (or Variational) Problems.
 - Very big problems, difficult for solvers
 - In any case, fundamental difficulties with integer variables.

Uncertainty (i)

- It comes in two flavors:
 1. It is possible to assign a probability (fuel prices, hydro conditions, demand evolution ...)
 2. It is not (mainly regulatory decisions)
- In the first case, it is possible to compute the equilibrium of the stochastic tree.
 - **Not the same than Montecarlo analysis**

Uncertainty (ii)

- Power systems operation is subject to relevant medium-term **uncertainty** factors:
 - Hydro conditions.
 - Demand.
 - Fuel prices.
- These uncertainties can be explicitly modelled by a **scenario tree**.



Uncertainty (iii)

- Each branch has associated probabilities w_{pb}
- Utilities maximize their profit expectation
 - Therefore, they must comply in each tree node

$$E[MC_u] = E[MR_u]$$

- Demand curve and power balance must be fulfilled in each branch.

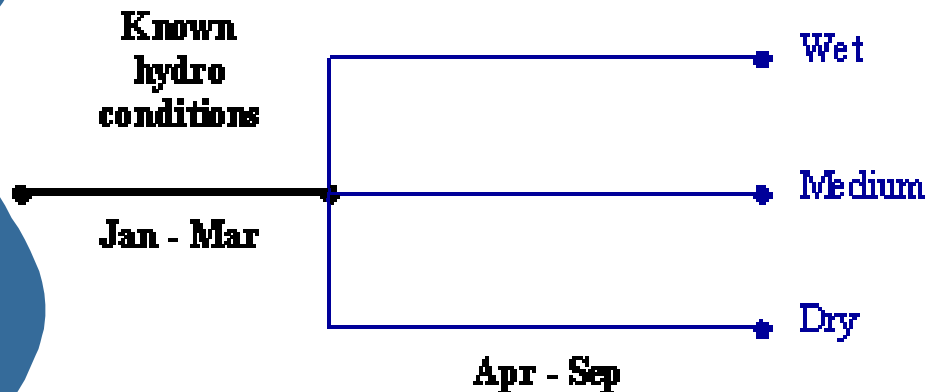
Uncertainty (iv)

- Then, **equilibrium equations** are shown to be equivalent to optimization model:

$$\begin{aligned} \min_{P_{upb}, D_{pb}} \quad & \mathbb{E} \left[\sum_u \left(\overline{C}_u \left(P_{upb} \right) \right) - U \left(D_{pb} \right) \right] \\ \text{s.t.} \quad & \sum_u P_{upb} = D_{pb} \quad : \lambda_{pb} \end{aligned}$$

Uncertainty (v) : Study case 1

Stochastic hydro conditions



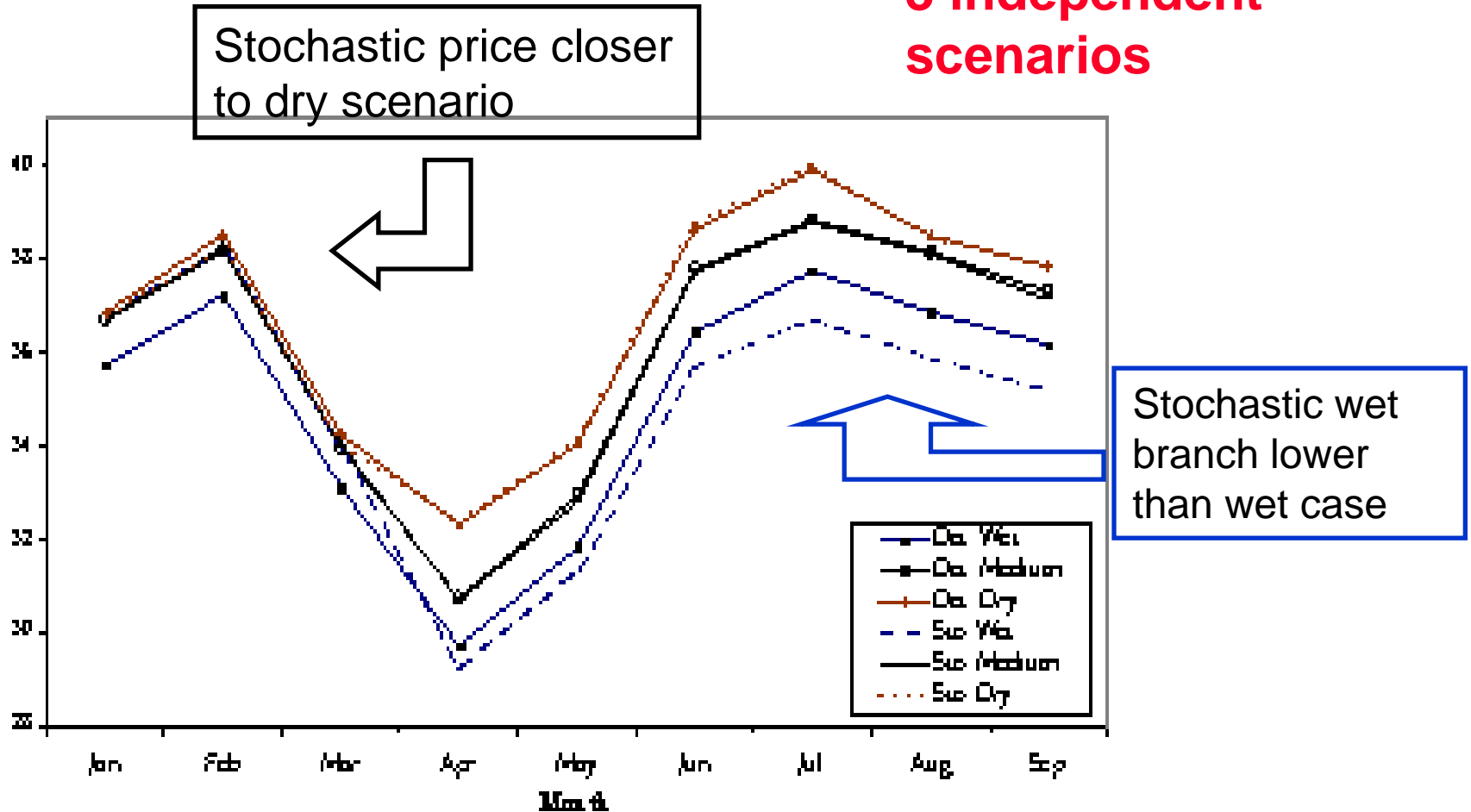
9 periods (months), 3 sub periods (working-days, Saturdays and Sundays) and 4 load levels.

Company	Nuclear	Coal	Gas	Hydro	Pumping
1	3459 (3)	5517 (15)	3259 (10)	4043 (7)	1431 (5)
2	3169 (5)	1167 (5)	4879 (11)	5421 (5)	2282 (2)
3	705 (1)	1888 (7)	1130 (3)	1518 (3)	216 (1)
4	155	1488 (5)	381 (1)	314 (1)	115 (1)
5	-	859 (5)	731 (2)	651 (1)	360 (1)
6	-	-	1518 (3)	-	-
7	-	117 (1)	1142 (3)	-	-

Installed power and number of plants

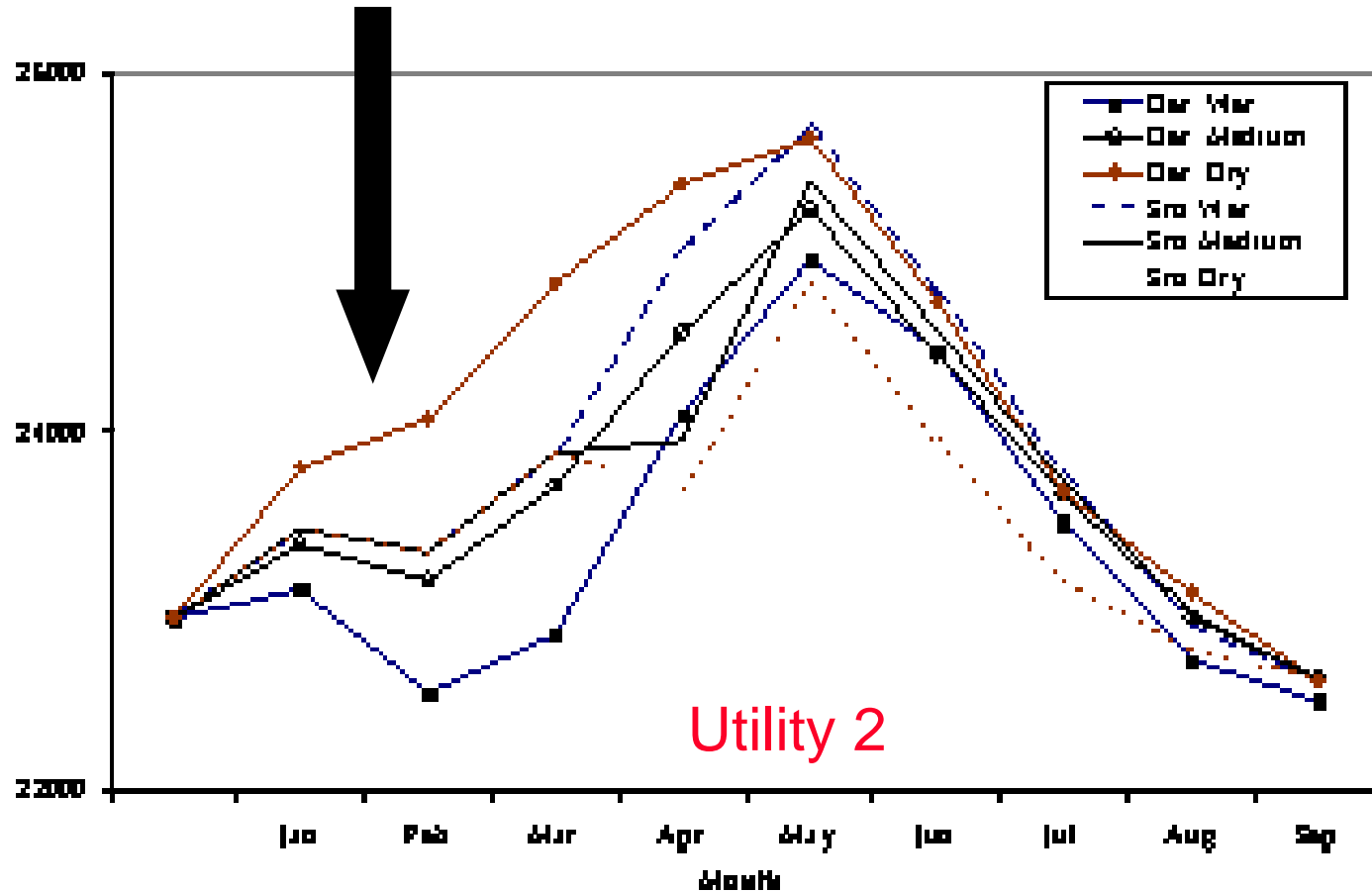
Uncertainty (vi) : Study case 2

Scenarios tree vs.
3 independent
scenarios



Uncertainty (vii) : Study case 3

Deterministic cases anticipate future inflows



Utility 2

Uncertainty (viii)

- In case of fuel prices, risk-neutral probabilities should be used.
- Not so clear for hydro inflows:
 - Rain uncorrelated with other markets: real probabilities.
 - But incomplete markets, anyway.
- And non-measurable uncertainty is, in any case, an open issue.

Network effects (i)

- There are many ways of conducting congestion management: capacity auctions, zonal pricing, ...
- Results can be very different.
 - **Equilibria may not exist**, for specific systems conditions and congestion management regulations
- Some of these procedures are easier to model than others.
 - In our approach, market splitting and some varieties of capacity auctions can be modelled.

Network effects (ii)

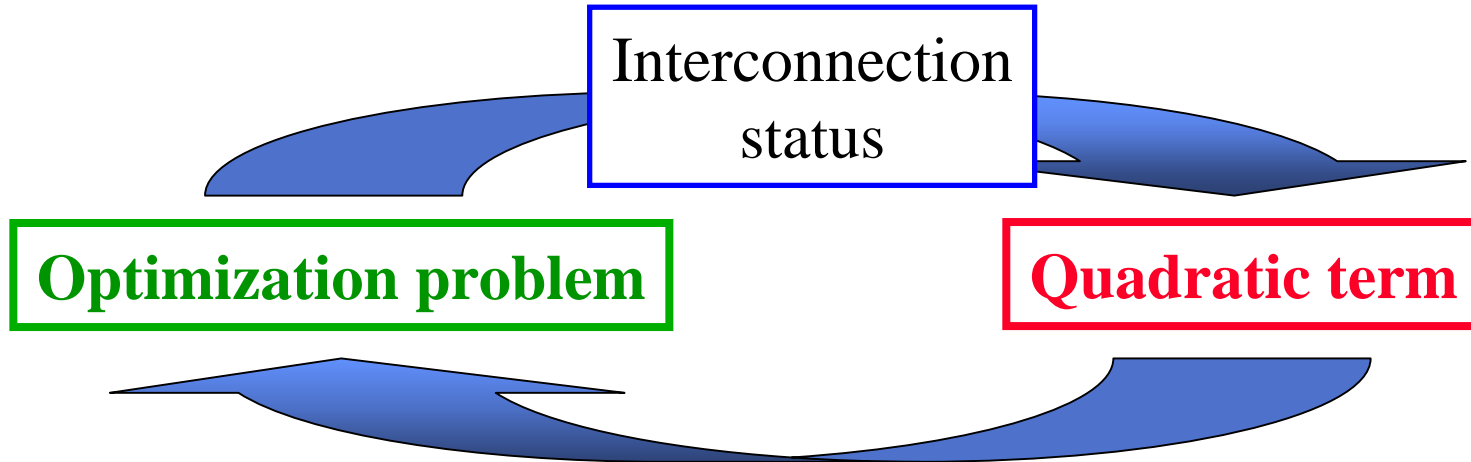
- To simulate “market splitting”

$$\begin{aligned} \min \quad & \sum_{a=\text{Syldavia, Borduria}; u=1, \dots, U} C_{au}(P_{au}) + Q(P_{au}, I) \\ \text{s.a.} \quad & \begin{cases} \sum_u P_{au} \pm I = D_a \\ I_{\min} \leq I \leq I_{\max} \end{cases} \end{aligned}$$

$$Q(P_{au}, I) = \begin{cases} \frac{1}{2} \sum_u \theta_u (P_{\text{Syldavia},u} + P_{\text{Borduria},u})^2 & \text{constrained off} \\ \frac{1}{2} \sum_u (\theta_{\text{Syldavia},u} P_{\text{Syldavia},u}^2 + \theta_{\text{Borduria},u} P_{\text{Borduria},u}^2) & \text{constrained on} \end{cases}$$

Network effects (iii)

Algorithm:



If it converges, a Nash equilibrium has been computed

Network effects (iv)

Data

- 2 areas
- Planning for 1 year. Periods: months.
- 6 load level blocks: weekend and working-day with levels peak, off-peak 1 and off-peak 2
- Demand: Area 1: 224757Gwh
Area 2: 45166Gwh
- Transmission capacity
 - Syl. to Bor.: 725Mw
 - Bor. to Syl.: 650Mw
- Water management between blocks of same period

Exported power (MW)

	hdy.pk	hdy.op1	hdy.op2	wdy.pk	wdy.op1	wdy.op2
01/01/2004	725	644	672	-650	357	350
01/02/2004	683	725	445	-650	211	341
01/03/2004	725	725	609	291	601	679
01/04/2004	207	695	214	398	328	527
01/05/2004	613	574	-8	464	230	115
01/06/2004	725	725	566	295	174	132
01/07/2004	725	725	725	559	329	318
01/08/2004	725	725	725	-24	342	328
01/09/2004	725	692	725	357	249	521
01/10/2004	725	725	673	634	622	321
01/11/2004	725	725	468	100	666	725
01/12/2004	725	725	646	-267	666	411

Proper modelling of hydro management is critical

Network effects (v)

- Open issues:
 - Absence/multiplicity of equilibria.
 - Difficulties of modelling certain constraint management procedures.
 - Many more conjectural variations (possibly including those in capacity markets).
 - Network and intertemporal effects can interact.

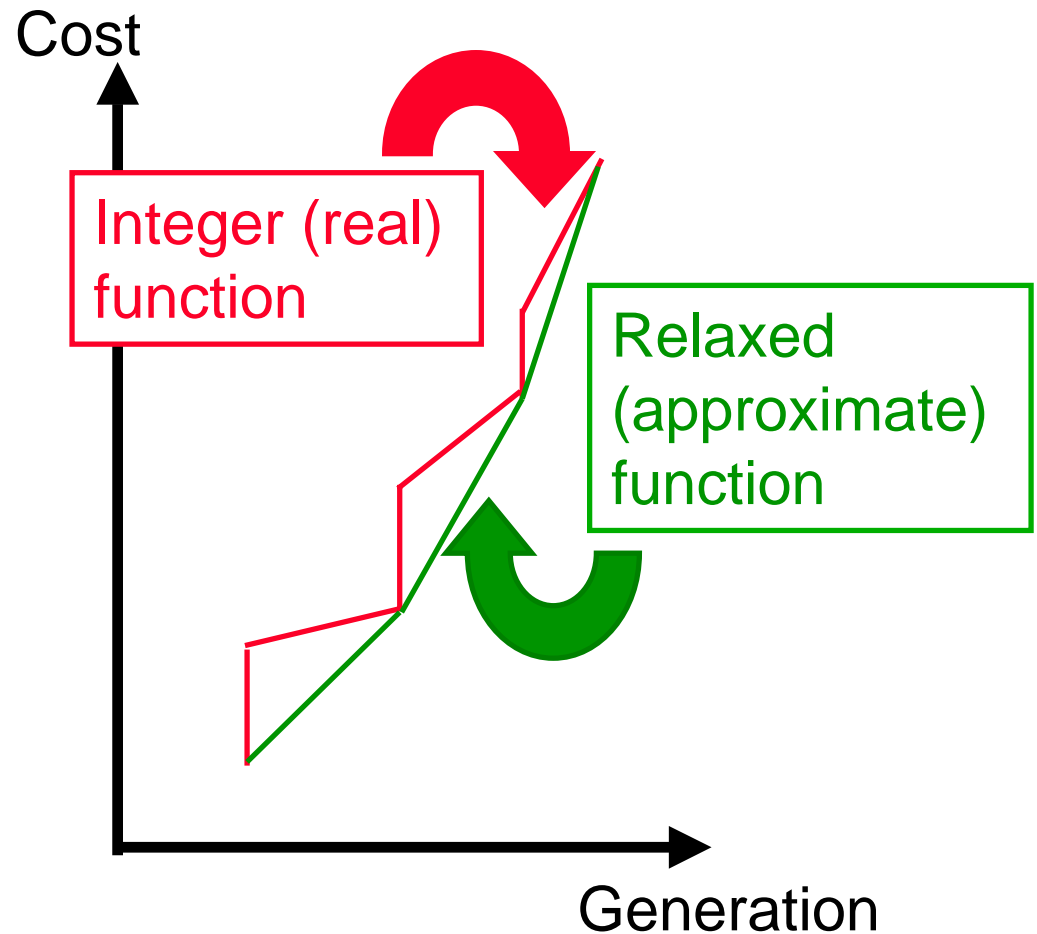
Hydro cascades

- In some systems, there are hydro plants belonging to different utilities in the same cascade.
- In these cases, not unusual to go to courts.
- Very few work on modelling these issues.
 - Very often, absence of “water markets”

Integer variables effects (i)

- Most models assume away integer variables, in order to have (among other things) convex cost functions.

In many studies, assumed cost function is the real cost function relaxing integer variables



Integer variables effects (ii)

1. Relaxed cost function leads to reasonable prices, but physically infeasible operation.
 2. Integer cost function leads to physically reasonable operation, but unreasonable prices.
- Both solution can be heuristically combined, but there is not rigorous basis to justify the procedure.
 - There are many equivalent ways to introduce integer variables in the multiperiod case.
 - The “most reasonable relaxed” function (the lower convex envelope of the cost function) is not generally a relaxed cost function, a in any case extremely difficult to compute.

Conclusion

- Present models are clearly unsatisfactory to address a number of issues.
- A lot of research work is needed, specially if specific forecasting of real systems is sought.