Towards anonymous undercollateralized loans

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ABSTRACT

Undercollateralized or unsecured loans are often considered impossible in fully decentralized and permissionless settings due to the lack of recourse for lenders if borrowers default. Past attempts at solving this problem often resort to semi-trusted system, in which one trusted party vouch for the credibility of the borrowers. In this paper, I consider the problem of pricing anonymous undercollateralized loans while allowing for the possibility of default. The only consequence to borrowers for defaulting is that subsequently, they only have access to over-collateralized loans for a given amount of time. I study all possible strategies borrowers may use and show the conditions for which always-honest strategy dominates all subgames, i.e., always-honest is a Subgame Perfect Nash Equilibrium. Importantly, even under conservative parameter assumptions, it is possible to charge reasonably high interest rate that also incentivizes honest behavior from anonymous borrowers. In addition, because the proposed system does not require consensus on external data, it is not subject to the oracle problem present in many decentralized applications. The paper may pave the way for experimenting with undercollateralized loans in fully decentralized settings, such as DeFi lending platforms that use public blockchain. Benefits of such a system include increasing liquidity provision and eliminating any biases against borrowers.

Keywords: Lending market design, Decentralized Finance, DeFi, Game Theory, Blockchain

JEL Classifications: D47, O31

1 Introduction

Fifteen years since Nakamoto (2008) introduced Bitcoin, decentralized finance (DeFi) has taken the world by storm. Nevertheless, DeFi still lacks one important feature: undercollateralized loans. Undercollateralized loans do not require collateral value to be greater than loan amount and do not expose borrowers to liquidation. In undercollateralized loans, if the value of the collateral drops below the value of the loan, borrowers may find it in their best interests to default, especially if there are no legal consequences. Thus, due to the anonymous nature of public blockchain, undercollateralized loans are considered impossible in DeFi (Aramonte, Doerr, Huang, and Schrimpf, 2022; John, Kogan, and Saleh, 2022; Harvey and Rabetti, 2023). This limitation not only hinders the potential of DeFi to facilitate liquidity for those in need but also exposes vulnerable borrowers to predatory lending driven by the threat of liquidation (Packin and Aretz, 2023). However, anonymity of borrowers helps eliminate bias. Evidence of discrimination in lending markets has been well documented in literatures across disciplines, e.g., Blanchflower, Levine, and Zimmerman (2003), Pager and Shepherd (2008), and Butler, Mayer, and Weston (2018).

Early efforts in designing semi-anonymous credit cards date back at least to Low, Maxemchuk, and Paul (1996). However, the system as designed in Low et al. (1996) is only semi-anonymous because it still involves a trusted third party to vouch for the borrower. The rise of DeFi has reignited research into anonymous loans. Noteably, *TrueFi* is one of the first platform that allows credit loans. However, *TrueFi* only allows institutional borrowers and requires extensive *Know-Your-Business* procedures. Thus, *TrueFi* operates more like a traditional bank that uses cryptocurrencies. Academic research into designing a credit system includes the works of Jain, Agrawal, Goyal, and Hassija (2019), Hassija, Bansal, Chamola, Kumar, and Guizani (2020), and Han, Chen, Qiu, Luo, and Qian (2021), among others. These designs often involve uploading users' encrypted credit profile onto the blockchain, then use *Zero-Knowledge Proof* to preserve the borrower's privacy while proving their credit score. Nevertheless, if lenders need to query the blockchain to learn about borrowers' credit score, they could simply ask the credit scoring agency via a platform such as *Plaid*.

In this paper, I reconsider the problem of designing an incentive-compatible system that allows for undercollateralized loans in fully decentralized settings. I formulate the problem as an infinitely repeated game between a lender and a borrower, both of whom are anonymous. The key difference in my system is that instead of allowing undercollateralized loans immediately, the lending platform permits them only after some given τ years of using overcollateralized loans. If a borrower defaults, he will have to do another τ years of overcollateralized loans before he can borrow undercollateralized loans again. Thus, in the worst-case scenario where there is no market for undercollateralized loans, my proposed system functions precisely like the existing system, which exclusively permits overcollateralized loans. Given τ and other observable parameters, the problem for the lender becomes choosing an interest rate r that incentivizes honest behaviors.[1] An incentive-compatible r should be high enough to incentivize lenders but also be low enough to incentivize payback from borrowers. The lower bound for r is trivially greater than the rate for overcollateralized loans. However, the upper bound is not as clear. The key result of this paper is a formal formula defining the upper bound of r for undercollateralized loans using only observable and measurable parameters. Relying only on observable and measurable parameters means the model is robust to adverse selection and non-truthful reporting. The model also functions in the presence of moral hazard, provided that the market is efficient and consistent with investors' profit maximization behavior. Once a lower bound and an upper bound are estimated, one can determine whether an anonymous undercollateralized lending market is currently feasible.

A key assumption to generate sustainable honest pay back from borrowers, and thus capital supply from lenders, is repeated borrowing. Readers who are familiar with game theory may recognize that this setup bears a resemblance to the infinitely repeated prisoners' dilemma game, in which a sustainable cooperation equilibrium can be achieved if players

[1] A trivial but infeasible solution is to set $r < 0$, that is, borrower gets paid to borrow.

anticipate repeated games. Bó (2005), and Dal Bó and Fréchette (2019) also provide empirical evidence for this result. Before applying repeated games to this paper's setting, it is important to recognize the cost of defaulting, even if there are no legal consequences. In the proposed system, borrowers cannot borrow undercollateralized loans again immediately after a default. They can only do so after some given number of periods. Hence, the main cost of defaulting is that borrowers must do overcollateralized loans again for τ periods. As explained in Packin and Aretz (2023), overcollateralized loans can be costly to borrowers because they can be liquidated at any time. Furthermore, overcollateralization may result in an opportunity cost to borrowers due to the returns they could have earned with a lower collateral requirement. The cost of losing access to undercollateralized loans is even greater if they default during their overcollateralized periods because of liquidation.

The key difference between the game considered in this paper and the classic prisoners' dilemma game is the dynamics of the payoffs. The classic prisoners' dilemma game often involves a fixed payoff matrix. However, since returns are stochastic and the actions of players can depend on ex-post returns, the classic prisoners' dilemma game cannot adequately model the interactions between anonymous lenders and anonymous borrowers. Solving for a game that involves an infinite number of possible paths presents a challenge. I solve this problem by relying on the assumption that the market is efficient, and thus asset returns are independently and identically distributed (i.i.d). Under the i.i.d. assumption, it is sufficient to solve for only an identical subgame (which starts and ends with undercollateralized loans) to determine the condition that a borrower will be honest.

Overall, I demonstrate the feasibility of anonymous and undercollateralized loans. Importantly, my results do not compete with other works on anonymous credit scoring systems. The progress and adoption of such credit scoring systems will work together with my proposed system to help bring true credit loans to DeFi.

The paper is structured as follows. Section 2 presents the main model and all assumptions. Section 3 validates the theoretical model with simulation results. I also use simulation to extend the main model to incorporate more complicated cases. Section 4 considers practical matters. I argue that the model can perform even better in practice because I impose rather conservative assumptions in the model. Finally, section 5 concludes and suggests future research.

2 Model

2.1 The setup

I consider an infinite game between a lender and a borrower in decentralized settings, e.g., a public blockchain. I now denote the parameters used in the model. Let r be the credit borrowing rate, τ be the number of periods (years) before credit loans are allowed, i be the discount rate or risk-free rate, c be the overcollateral borrowing rate, u be the unobservable borrower's return from using the loan, y be the return the borrower gains from holding his collateral assets (y can be equal to c), k be the collateral-to-loan ratio for overcollateralized loans ($k > 1$), l be the collateral-to-loan ratio for undercollateralized loans ($l \leq 1$), and ρ_t be the stochastic collateral asset's returns. Following the convention, $\bar{\rho}$ denotes the expected collateral assets' return. The only assumption regarding the collateral asset's returns is that they are independently and identically distributed, which follows from the Efficient Market Hypothesis (EMH). Thus, the model is robust to the specific form and type of the asset's distribution. In the numerical simulation, I follow the convention and assume a log-normal distribution. Given any distribution for the asset's returns, one can compute the probability of $1 + \rho_t < 1 + r$. This probability will be important in computing the expected payoff for the borrower. The idea here is that if the cost of loan, r, is higher than the asset's return, ρ_t, then it is better to default. Let $F = F(1+r) = Pr(1+\rho_t < 1+r)$ be the aforementioned probability. Regarding the borrower's return u, I assume that the borrower always attempts to maximize his payoff, regardless of the type of loan he receives. Thus, u is independent of the type of loan. This assumption on u will be relaxed later to allow for the moral hazard problem.

Following Rivera, Saleh, and Vandeweyer (2023), I assume that the lender will lend if the expected interest rate that he receives exceeds his other options. The borrower will borrow every period and will have to honor his debt if he borrows overcollateralized loans, otherwise he may choose to default. Undercollateralized loans require a collateral ratio $l \leq 1$ and cannot be liquidated early by the lender (this helps avoid the problem with predatory lending, as pointed out in Packin and Aretz, 2023).

2.2 Payoff functions of possible strategies

Before defining the payoff functions, it is important to review the strategies that players can choose. If the borrower uses overcollateralized loans, he is better off repaying the loan to avoid liquidation. The success of overcollateralized loans in practice provides strong evidence for this. However, the focus of this paper is undercollateralized loans. If the borrower uses undercollateralized loans, then he has a total of four possible strategies: always paying back the debt (*Always-Honest*, or H strategy), always defaulting and creating a new address (*Always-Selfish*, or S strategy), choosing to pay back only when paying back gives higher payoff (*Mixed*, or M strategy), and choosing to be honest for some fixed N number of periods (*Mixed-Fixed*, or MF strategy). In the M strategy, the borrower will repay the debt only if the value of the collateral is higher than the total cost of debt, which is the principal plus any interest. As a note, L and B subscripts are used to denote the lender's and the borrower's payoff, respectively. Formally, an M-type borrower will repay if and only if:

$$H_B > S_B \Leftrightarrow l(1 + \rho_t) > 1 + r$$

Before proceeding, I offer two propositions that are useful to reduce the number of strategies that need to be considered, Proposition 1.A and 1.B.

Proposition 1.A. *Let N^* denote the number of honest periods that maximize borrower's* *payoff. N^* can only be either $N^* = 0$ or $N^* = \infty$.*

Proposition 1.B. *The M strategy's expected payoff is at least as high as the S strategy's.*

In Proposition 1.A, the first case with $N^* = 0$ is equivalent to the S strategy, in which the borrower never acts honestly; the second case with $N^* = \infty$ is equivalent to the H strategy, in which the borrower always acts honestly. Combining Proposition 1.A and 1.B, I only need to consider the borrower's payoffs for the H strategy and the M strategy to derive the condition for honest payback.

Payoffs in the case of overcollateralized loans are simple. The lender will receive his principal P back plus interest c:

$$B_L = P(1 + c)$$

The borrower will pay back the principal and receive his return u less interest c:

$$B_B = P(u - c)$$

In the case of undercollateralized loans, the payoffs for both the lender and the borrower depend on whether the borrower honor his debt. If the borrower pays back the debt, the lender will receive his principal P back plus interest rP:

$$H_L = P(1 + r)$$

An honest borrower will receive:

$$H_B = P\left(u - r + l\rho_t\right) + \left(k - l\right) Py \left(1 + \rho_t\right)$$

In case the borrower defaults on his undercollateralized loans, the lender will get the collateral Pl, which has value $Pl(1 + \rho_t)$. Thus, the lender's payoff will be:

$$S_L = Pl(1 + \rho_t)$$

If the borrower defaults, he will receive:

$$S_B = P\,(u + 1 - l) + (k - l)\,Py\,(1 + \rho_t)$$

In the first term of S_B, observe that if the borrower defaults, he will lose a value of Pl, which is the historical cost of the collateral for the undercollateralized loans, regardless of the current value of the collateral. I account for the loss in this way because in the event of default, the borrower will forfeit his collateral entirely to the lender. The benefit that the lender receives from liquidating the collateral depends on the return of the asset, as denoted in S_L by the term $(1 + \rho_t)$. To better illustrate this difference, consider the case with $P = 10,000$, $r = 10\%$, and $l = 1$. In this example, the borrower pledges a collateral worth $10,000$ initially to obtain a loan of $10,000$. On maturity, the borrower will have to pay $11,000$ to the lender to get his collateral back. If the collateral's value drop significantly to 200 (a drop of 98%), the borrower defaults and forfeits his collateral. The current value accounting stipulates that the borrower only lost 200 while the historical cost accounting records that he lost $10,000$. Clearly, the current value accounting does not make sense here. It is important to differentiate this case with the recent new rule by the *Financial Accounting Standard Board* (FASB) to record crypto holdings at fair value.[2] The key difference is that FASB's accounting rule applies to holdings, while in the case of forfeiting collateral, the borrower does not hold the collateral anymore. In my model, the FASB's rule is applied to the lender, who is the holder of the assets in case of default.

In the second term, observe that absence the overcollateralization requirement, the borrower may use the extra collateral assets to earn the return y. The borrower can do this by lending out the extra assets to overcollateralized loans, in which case $y = c$. This extra return means that even without considering liquidity costs of overcollateralized loans, these loans impose an opportunity cost on borrower through y. The benefit from avoiding the overcollateralization requirement is captured by the term $(k - l)\,Py\,(1 + \rho_t)$, which is present in

[2] https://www.fasb.org/page/getarticle?uid=fasb_Media_Advisory_12-13-23

both H_B and S_B.

In this paper, I consider the problem of choosing the credit interest rate r, given all other parameters. Let $H(r) : \mathbb{R} \rightarrow \mathbb{R}$ and $M(r) : \mathbb{R} \rightarrow \mathbb{R}$ be the total expected payoff from H-strategy and M-strategy, respectively. The payoff function $H(r)$ is given as follows:

$$H(r) = \mathbb{E}\left[\sum_{t=0}^{\infty} H_B \mathrm{e}^{-it}\right] \tag{1}$$

where

$$H_B = P\left(u - r + l\rho_t\right) + \left(k - l\right) Py \left(1 + \rho_t\right)$$

For $M(r)$, indicator functions are needed as $M(r)$ depends on borrower's actions, not only in period t, but also in the last τ periods. The function can be described formally as follows:

$$M(r) = \mathbb{E}\left[\sum_{t=0}^{\infty} \left(\mathbb{1}_U \left(\mathbb{1}_H H_B + (1 - \mathbb{1}_H) S_B\right) + (1 - \mathbb{1}_U) B_B\right) \mathrm{e}^{-it}\right] \tag{2}$$

where

$$\mathbb{1}_H = \begin{cases} 1, & \text{if } \rho_t > r \\ 0, & \text{otherwise} \end{cases}$$

$$\mathbb{1}_U = \begin{cases} 1, & \text{no default in the last } \tau \text{ periods} \\ 0, & \text{otherwise} \end{cases}$$

$$H_B = P\left(u - r + l\rho_t\right) + \left(k - l\right) Py \left(1 + \rho_t\right)$$

$$S_B = P\left(u + 1 - l\right) + \left(k - l\right) Py \left(1 + \rho_t\right)$$

2.3 Participation condition

The borrower should always prefer undercollateralized loans because he has the option to default. The borrower will follow H-strategy if and only if $H(r) > M(r)$. For the lender, if he expects the borrower to default, he will not participate in undercollateralized lending. The lender should be willing to supply capital if he can earn more than some minimum return and he can be assured of honest behavior from the borrower. Given that the lender can earn return c through overcollateralized lending, the conditions for the lender's participation are: (1) $r > c$, and (2) $H(r) > M(r)$. Overall, the conditions for a feasible anonymous and undercollateralized loan market are:

$$r > c \tag{3}$$

$$\mathbb{E}[H(r) - M(r)] > 0 \tag{4}$$

The conditions appear deceptively simple. The main challenge lies in computing $\mathbb{E}[H(r)-M(r)]$. The naive approach is to compute $H(r)$ and $M(r)$ then take the difference, or to compute the difference $\mathbb{E}[H(r) - M(r)]$ directly. Since the M-type borrower conditions his action on ex-post stochastic returns, computing $M(r)$ requires knowing the joint distribution function of the returns for an infinite number of periods, which is infeasible. A better approach is to focus on the difference $\mathbb{E}[H(r) - M(r)]$ but only up to the *decision node*. A decision node is a period in which the borrower gets access to undercollateralized loans and can decide whether to default. Because this is an extensive form game, one can divide this infinitely repeated game into many identical subgames with the starting node for each subgame being the decision node. The reason why subgames defined in this way are identical follows directly from the assumption that the asset return, ρ_t, is identically and independently distributed. Intuitively, at each decision node, the borrower faces the exact same problem of deciding whether to default because the borrower can disregard any earlier returns. Recall that the borrower is given the option to default only in case of undercollateralized loans. In addition, he can only default once every τ periods. An example subgame for $\tau = 2$ with all possible actions for the M-type borrower is visualized in Figure 1. For $\tau = 2$, one needs to consider up to five periods to exhaust all possible interactions. The naive approach of considering only $\tau + 1$ periods is not enough because it fails to account for the case when the borrowers who honor their debts still get to choose, while those who default do not (because defaulted borrowers can only do overcollateralized loans for the next τ periods). Another approach that does not work is to apply the options pricing framework. This is what has been used by *BlockScience* team to evaluate the *THORChain* platform's risks.[3] The report shows that secured lending without liquidation is akin to selling an American call option. The key difference between my model and *THORChain*'s is the τ overcollateralization periods. *THORChain* does not impose any restriction besides the initial overcollateralization. The platform then allows borrowers to pay back their debt at any time without liquidation. Given that *THORChain* does not allow undercollateralized loans, its settings do not apply to this paper.

As shown earlier, because the M-type borrower's decision depends on the collateral's returns, the payoff he receives is conditional. That is, for any given period, M-type borrower's expected payoff from being honest is $\mathbb{E}[H_B \mid l(1+\rho_t) > 1+r]$. The M-type borrower's expected payoff from being selfish is $\mathbb{E}[S_B \mid l(1+\rho_t) < 1+r]$. In case being honest pays more, the difference between the H-type borrower and the M-type borrower is $\mathrm{D}_{HH} = \mathbb{E}\left[H_B - M_B \mid l(1+\rho_t) > 1+r\right] = 0$, since both borrowers choose the same action. In case defaulting pays more, the difference between the H-type and the M-type borrower is $\mathrm{D}_{HS} = \mathbb{E}\left[H_B - M_B \mid l(1+\rho_t) < 1+r\right]$. Finally, anytime a borrower defaults, his next τ periods' payoffs are deterministic since he can only do overcollateralized loans and has no choice but to pay back. Hence, M-type borrower's actions after defaulting is not conditional: $\mathrm{D}_{HB} = \mathbb{E}\left[H_B - B_B\right]$. I now present the main result of this paper, the condition for which $\mathbb{E}[H(r) - M(r)] > 0$, which guarantees that H-strategy dominates M-strategy, rendering the threat

[3] https://hackmd.io/@blockscience/H1Q-erh_n/%2FgedrnocFRxOWWgT8TXJ-sw%23Lending-as-an-American-Call-Option

of defaulting not credible.

Proposition 2. $\mathbb{E}[H(r) - M(r)] > 0 \Leftrightarrow$

$$\frac{F(\mathrm{D}_{HB}\mathrm{e}^{(-2\tau-1)i}(1-F)^{\tau+1})}{(\mathrm{e}^i - 1 + F)(\mathrm{e}^i - 1)} + \frac{F\left((-\mathrm{D}_{HB} + \mathrm{D}_{HS})\,\mathrm{e}^{-i(\tau+1)} - \mathrm{D}_{HS}\mathrm{e}^{-i\tau}\right)(1-F)^{\tau+1}}{(\mathrm{e}^i - 1 + F)(\mathrm{e}^i - 1)}$$
$$+ \frac{F(\mathrm{D}_{HS}\mathrm{e}^i + \mathrm{D}_{HB} - \mathrm{D}_{HS}\mathrm{e}^{-i\tau}\mathrm{D}_{HB})}{(\mathrm{e}^i - 1 + F)(\mathrm{e}^i - 1)} > 0 \tag{5}$$

where

$$\begin{aligned}
\mathrm{D}_{HS} &= \mathbb{E}\left[H_B - S_B \mid l(1+\rho_t) < 1+r\right] \\
&= \mathbb{E}\left[P(l(1+\rho_t) - (1+r)) \mid l(1+\rho_t) < 1+r\right] < 0 \\
\mathrm{D}_{HB} &= \mathbb{E}\left[H_B - B_B\right] \\
&= P\left(c - r + l\bar{\rho}\right) + (k-l)\,Py\,(1+\bar{\rho})
\end{aligned}$$

Proposition 2 gives the explicit formula for the dominance of the H-strategy. If the credit interest rate $r > c$ satisfies Ineq. 5, it ensures that the expected payoff for the H-strategy is higher than the expected payoff for the M-strategy. This means that it pays more to always be honest and pay back the debt. Note that solving 5 involves computing the conditional expectation. Hence, a numerical solver is needed because the distribution of the collateral asset's return, e.g., normal, often does not have a closed form formula.

2.4 Comparative statics

As indicated in Table 1, the only parameter that is not observable to the lender is the borrower's return u. If u is required in order to price undercollateralized loans, then one needs to have a truthful reporting mechanism or the system cannot work. Fortunately, it is straightforward to show that the inequality in 5 does not depend on u. Observe that u drops out of both D_{HS} and D_{HB}. Thus, $\mathbb{E}[H(r) - M(r)]$ does not depend on u. Nevertheless, it is important to note that this result relies on the assumption that u is the optimal return for the borrower, regardless of the type of loan he receives. If the borrower receives a return conditional on the type of loan (undercollateralized vs. overcollateralized), then one must know the return on the borrower's loan. This problem can occur under moral hazard, in which the borrower invests differently depending on the type of loan. Modeling moral hazard involves trading off tractability, thus this problem is studied using simulation in Section 3.2.2.

Observe that the loan amount P serves as a scaling factor in the left-hand side of 5, meaning that as long as the left-hand side is positive, a higher loan amount only increases the incentive to be honest. This is important because if the formula only works for small P, the market cannot scale.

The overcollateral interest rate c only appears in D_{HB}. Differentiate the LHS of 5 w.r.t D_{HB} yields

$$\frac{F\left(\mathrm{e}^{-i(\tau+1)}(1-F)^{\tau+1}-1\right)\left(\mathrm{e}^{-i\tau}-1\right)}{\left(\mathrm{e}^{i}-1+F\right)\left(\mathrm{e}^{i}-1\right)} \geq 0, \quad \forall i > 0$$

Thus, $\mathbb{E}[H(r)-M(r)]$ increases with D_{HB}. Intuitively, the advantage of always being honest is avoiding secured loans. Therefore, the larger the difference between the honest payoff in undercollateralized loans and the overcollateralized loans payoff, the larger the incentive to be honest. Observe that the overcollateralized loans' interest rate c appears only in D_{HB} and is positively correlated with D_{HB}. This correlation indicates that increasing c also increases the honest's advantage. This is expected because higher c makes borrowing overcollaterally more expensive, which should encourage borrowers to refrain from defaulting in order to keep the option of borrowing undercollaterally. Other parameters also have the expected sign. A higher credit interest rate r reduces the payoff for honest borrowers since they will have to pay higher price for the credit loans. Selfish borrowers are not affected because they can default. Thus, a higher r reduces the incentive to be honest. The interest borrowers can receive from not having to lock up more assets, y, helps increases the incentive to be honest since honest borrowers can keep receiving y. In the model calibration and simulation section, I assume that $y = c$, meaning that borrowers can become lenders themselves for overcollateralized loans using the extra capital from not having to borrow overcollaterally.

2.5 Incorporating market clearing condition

As specified in Table 1, the only parameters that need to be identified are τ and r. Previous analysis took τ as given and focused on choosing r (recall from the earlier analysis that P simply scales the payoffs without affecting $\mathbb{E}[H(r)-M(r)]$). I also did not include the market clearing condition to emphasize the possibility of having anonymous undercollateralized loans at least for some players, even if not all players get what they want, i.e., the market does not necessarily clear. Nevertheless, incorporating the market clearing condition is straightforward and can help relax the assumption of known τ, albeit at the costs of requiring known forms of capital supply and demand functions. Given the capital supply and demand functions, the market clearing rate, r^*, can be obtained. Assuming τ is not known, given the market clearing interest rate, the condition specified in Proposition 2 can then be used to determine τ.

Following Rivera et al. (2023), the capital supply curve, $\mathcal{S}(r)$ can be thought of as the distribution function for the probability of having the lenders' outside interest rate less r. Similarly, the capital demand, $\mathcal{D}(r)$ can be thought of as the distribution function for the probability that borrowers' external interest rate is higher than r. Specifically, $\mathcal{S}(r)$ and $\mathcal{D}(r)$ can have the following forms:

$$\mathcal{S}(r) = 1 - \mathrm{e}^{-z_{\mathcal{S}} r} \tag{6}$$

$$\mathcal{D}(r) = \mathrm{e}^{-z_{\mathcal{D}} r} \tag{7}$$

where

$$z_f = \text{controls the rate sensitivity for function } f$$

$$r = \text{the anonymous credit interest rate}$$

Solving $\mathcal{S}(r) = \mathcal{D}(r)$ gives the market clearing interest rate r^*. Let $\bar{r}$ be the interest rate that satisfies the incentive-compatible condition, as specified in Proposition 2. If $r^* \leq \bar{r}$, an anonymous undercollateralized loan market is feasible and is cleared. If $r^* > \bar{r}$, the market will not clear as not enough lenders will be willing to lend at $\bar{r}$ to satisfy the demand. Nevertheless, some lenders may still find $\bar{r}$ attractive enough as $\bar{r}$ is at least feasible, i.e., guaranteeing honest repay.

3 Model calibration and simulation study

In this section, I calibrate the model and use Monte Carlo simulation to conduct sensitivity analysis and extend the model to relax some of the initial assumptions.

3.1 Model calibration

The discount rate i is assumed to be 2%, following the target inflation rate by the Federal Reserve. The loan amount P is set at $10,000$. The parameter τ is set at 3, which means that a borrower can only use undercollateralized loans after at least 3 years of borrowing overcollaterally. The overcollateral borrowing rate, c, is assumed to be 2%. As discussed in Section 2.4, higher c reduces the incentives to pay back loans. Thus, setting $c = 2\%$ is conservative because the borrowing rate for stablecoins on popular DeFi platforms is often higher than 2%, with the average being around 5%.[4] The return that undercollateralized loans borrowers can gain from their extra capital, y, is assumed to be equal to c. The collateral-to-loan ratio for overcollateralized loans $k = 2$, and the ratio for undercollateralized loans $l = 1$, i.e., the loan is fully-backed initially, with the possibility of being undercollateralized if the collateral decreases in value. The distribution for the collateral asset returns is estimated using the *S&P500*'s annual returns, which are collected from *CRSP*. The reason for choosing *S&P500* instead of a cryptocurrency like *Bitcoin* is because of data limitation. *S&P500* has a much longer history, while *Bitcoin* currently only has a maximum of 14 years of data.

[4] https://dune.com/bg_research/defi-rates

In addition, the returns of *Bitcoin* for the early years are not reliable. Regardless, given its extraordinary returns, using *Bitcoin*'s returns would only make the results stronger, i.e., $\mathbb{E}[H(r) - M(r)]$ is larger if *Bitcoin* is used as collateral. The distribution for the collateral asset is assumed to be log-normal. Note that the model is robust to the specific type of asset's distribution. The decision to use log-normal distribution is purely for conventional purpose and ease of calculation. Using *S&P500*'s annual return from 1925 to 2023, the estimated expected return $\bar{\rho} \approx 11.77\%$. Under these assumptions, the maximum interest rate for undercollateralized loans can be obtained from solving $\mathbb{E}[H(r) - M(r)] = 0$ using the result from Proposition 2. The result is $r \approx 10.80\%$, which is more than 5 times as high as the assumed c. Given that $r \approx \rho$, an investor who is interested in holding the collateral asset should be interested in providing capital in this undercollateralized lending market because he can earn similar returns with reduced volatility. If the formula is correct, a rate slightly above 10.80% should cause $\mathbb{E}[H(r) - M(r)] < 0$, while any rate below 10.80% should cause $\mathbb{E}[H(r) - M(r)] > 0$. Simulation confirms this result.

Figure 5 depicts how interest rate, r, changes against key parameters. The plots for u, c, and y are all consistent with the analysis in 2.4. Observe that while r increases with τ, the rate of change exhibits diminishing rate. Only the first few τ contributes to raising r. Beyond $\tau = 10$, the effect of τ on r is minimal. This indicates that τ can be chosen by lenders arbitrarily without significantly affecting the r. For the discount rate i, the maximum r decreases in i, suggesting that the model works better when borrowers are more patient. This behavior is consistent with standard results in the infinitely repeated prisoners' dilemma (Bó, 2005).

The next section describes the simulation process and extends the model to more complicated cases.

3.2 Simulation results

The simulation study is conducted following the *cadCAD* framework.[5] *cadCAD* stands for *Complex Adaptive Dynamics Computer Aided Design*, which is a Python package for studying complex systems using simulation. *cadCAD* has been applied in practice to study the risks of DeFi platforms, such as *THORChain*. Another reason I choose *cadCAD* is because it is open source, making it easy for readers to replicate the results of this paper.

The simulation employs the payoff functions as specified in 2.2. As discussed in 3.1, the return of the collateral asset is modeled after the *S&P500* and is assumed to follow a log-normal distribution. Specifically, at the beginning of each period, I sample from a normal distribution with mean ≈ 0.094 and standard deviation ≈ 0.191. I then take the exponential of this sample to get a sample for $1 + \rho_t$. The sampled ρ_t is then used to decide whether the M-type borrower will default and to compute all players' payoffs. All other parameters are as identified in 3.1.

One important result from Proposition 2 is that the honest advantage does not depend on u, which is unobservable. Figure 2 plots the maximum incentive-compatible interest rate $\bar{r}$ ($H(\bar{r}) = M(\bar{r})$) against borrower's return, u. The blue line, $\bar{r}$, results from simulation. To get the simulated $\bar{r}$, for each u, I run the simulation assuming $r = 9\%$ to $r = 15\%$ with 0.1% incremental steps and keep the r such that the absolute difference $\mid \mathbb{E}[H(r) - M(r)] \mid$ is minimized. The values for u range from $u = 1\%$ to $u = 29\%$, with 1% incremental step. For each set of parameters, I run 1,000 simulations for a total of $1,800,000$ simulations. Each simulation is run for 100 periods. The numerical r is obtained by numerically solving the root of 5.

The simulation approach has some randomness, and the numerical procedure inevitably contains numerical errors (plugging in $r = 10.8\%$ yields the smallest difference in $\mid \mathbb{E}[H(r) - M(r)] \mid$ for all u). Nevertheless, it can be seen from Figure 2 that both approaches produce similar results, confirming Proposition 2. Importantly, there is no clear pattern when plotting

[5] https://cadcad.org/

r against u, confirming that the incentive-compatible credit interest rate, r, is independent from the unobservable u.

Having confirming the theoretical results, I now use simulation to extend the model to include cases that are too complicated for the analytical approach.

3.2.1 Including non-zero probability of failure. As a practical matter, borrowers may fail to pay back not because of their selfishness, but because of unwanted failure. That is, the borrowers cannot pay back even if they want to. This situation hurts both the lenders and the borrowers. Because the market is anonymous, there is no way for the lenders to know whether the default was because of selfishness or failure. Hence, any default is treated similarly, and the defaulted borrowers must start the τ overcollateralized loans periods again. If a higher failure rate reduces the honest advantage, then the proposed model may not work during times of distress. Incorporating a non-zero failure rate creates too many possible paths to consider for the analytical approach, hence I use simulation. Assuming the failure is unwanted, the failure rate must be independent from the type loans taken. That is, borrowers face the same failure rate in both over- and undercollateralized loans.

Figure 3 plots the simulated maximum incentive-compatible rate, $\bar{r}$, against the failure rate. It is clear that a higher failure rate increases the honest advantage, as evidenced by the higher maximum interest rate r that the lenders can charge. This result indicates that the proposed model may perform even better when borrowers face a non-zero probability of failure. The intuition for this is borrowers face much harsher consequences when they default their overcollateralized loans. This is similar to the problem pointed out in Packin and Aretz (2023) regarding predatory lending. Borrowers who get early liquidated lose all their collateral before they have the chance to pay back. Because of this, borrowers will want to avoid overcollateralized loans if they face higher default rate so they may default without consequences when they have to.

3.2.2 Moral hazard. The previous analysis assumes that failure is unwanted and thus is independent of loan type. However, without overcollateralization, borrowers may have moral hazard. That is, they may invest in riskier projects when they take out undercollateralized loans. Moral hazard can be detrimental to the lending market because it increases the cost to the lender, forcing the lender to raise the cost of loans (Igawa and Kanatas, 1990; Acharya and Viswanathan, 2011; Ioannidou, Pavanini, and Peng, 2022). In the proposed model, increasing the interest rate also encourages borrowers to default, which may render the model infeasible. Nevertheless, the specific form of moral hazard is important to consider. If all borrowers are rational and the market is efficient, then all investment should have similar risk-adjusted returns. Skeptical readers may question the spread between bond and stock returns à la the "equity premium puzzle". This puzzle can be rationalized to be consistent with optimization behavior by the works of Benartzi and Thaler (1995), Bansal and Coleman (1996), Palomino (1996), and Holmström and Tirole (1998), among others.

Given that all investments must have similar risk-adjusted returns, I model moral hazard as follows. Borrowers will invest in a risk-free project that pays a return of $y = c$ if they take out overcollateralized loans. If they take out undercollateralized loans, borrowers will invest in a risky project that pays a return of $y_M > y$ if successful but pays nothing if it fails. The probability of success for the risky project is such that $\mathbb{E}[y] = \mathbb{E}[y_M]$ to be consistent with the Efficient Market Hypothesis and investors' optimization behavior. Without loss of generality, let $y_M = wy$, with $w > 0$. The probability of failure for the risky project $q = 1/w$. Setting $r = 10\%$ and $w = 4$ (the risky project pays 4 times as much as the risk-free project) with other parameters as specified in 3.1, I use simulation to find $\mathbb{E}[H(r) - M(r)]$ across different u ranging from $u = 10\%$ to $u = 100\%$ with an incremental step of 10%. I once again focus on u because only u is unobservable. If the presence of moral hazard makes the honest advantage $\mathbb{E}[H(r)-M(r)]$ dependent on u, then the whole model has an identification issue and cannot work in practice. Figure 4 plots simulated $\mathbb{E}[H(r) - M(r)]$ against u for four independent simulation runs. There is no discernible pattern in the relation between

$\mathbb{E}[H(r) - M(r)]$ and u in all four simulations.

4 Practical considerations

So far the model has imposed rather conservative assumptions. Many of the assumptions are in place to test whether the proposed model can work without widespread adoption and support from the existing infrastructure. In this section, I consider how the proposed system can work in practice and argue that it may work even better than in theory.

4.1 Oracle problem

The oracle problem, in the context of decentralized system, refers to the challenge of ensuring the validity of data that is external to the blockchain (John et al., 2022, Rivera et al., 2023). Any data that is not internally generated by the blockchain cannot inherit the blockchain's security, e.g., credit rating or exchange rate. The main problem with having an oracle is the amount of trust involve. The oracle essentially decides the outcome of the contract. Thus, the oracle can be bribed by either party to lie. This is particularly problematic because the losing party has more incentives to bribe the oracle. For example, consider a bet of some X value between Alice and Bob, and the outcome of the bet is reported by an oracle. Assuming the correct result is that Alice wins and Bob now has to pay her X. Bob is willing to pay the oracle up to X to get the result in his favor. To compete with Bob, Alice has to sacrifice her entire winning value. Thus, the oracle problem affects players unequally and can render any decentralized system no longer trustless. This problem essentially affects all current DeFi lending platforms because they all need to query for current exchange rate to determine contracts' outcomes (Qin, Zhou, Gamito, Jovanovic, and Gervais, 2021, Gudgeon, Werner, Perez, and Knottenbelt, 2020, Al-Breiki, Rehman, Salah, and Svetinovic, 2020).

I claim that my model is robust to the oracle problem. This is because the model does not require both lenders and borrowers to agree on the current exchange rate. Players are free to follow their source of data. The choice to pay back depends entirely on the borrower's best interest, hence there is no reason to bribe a price oracle.

4.2 No reuse of assets by defaulted borrowers

In the main model, I allow borrowers to freely send their remaining assets to a new account after defaulting. Thus, borrowers who defaulted can easily rebuild their overcollateralization history. This may not be true in practice because assets on public blockchain, such as *Bitcoin*, can be marked. Tracking assets can be helpful in preventing illicit activities. For example, firms like *Chainalysis*[6] and *Tether*[7] frequently monitor public blockchains and track illegal activities. Once marked, selfish borrowers may find it hard to use their assets in other platforms and services, which can deter them from defaulting in the first place. Even without the adoption from other services, because borrowers cannot reuse assets that were associated with their old account, they will have to acquire new assets to use as collateral, which can be costly.

4.3 Reputation system

Besides allowing free reuse of collateral, I also do not consider any credit rating or reputation system. Using the proposed system with any anonymous credit rating system should strengthen the incentives to be honest. Perhaps the best example of a working reputation system comes from the dark web merchants. The dark web presents a similar setting to an anonymous loan market in which the merchant is the borrower and the buyer is the lender. The buyer sends money to the merchant for some goods and/or services without any guarantee of fulfillment. While some merchants may use an escrow system to enhance trust, doing so simply shifts the point of trust to the escrow. Trust in the dark web is an interesting topic of research on its own. However, perhaps due to data limitations, research in this area

[6] The Chainalysis 2024 Crypto Crime Report
[7] Tether Freezes $225M Linked to Human Trafficking Syndicate Amid DOJ Investigation

remains scarce. Laferrière and Décary-Hétu (2022), and Koutrouli and Tsalgatidou (2012) provide a glimpse into the workings of illegal online marketplaces. Obviously there is no buyer protection or any possibility of recourse for the buyer in case the seller disappears with the money. In these markets, sellers often attempt to instill trust by showing past customers' reviews, appearing professional, and insisting that they are here for the long term to make sustainable profits. Applying to anonymous lending market, borrowers may appeal to lenders by showing their history of repaying undercollateralized loans, or they may give more details on how they plan to use the funds. Besides reputation, trust can be built using a *Web-of-Trust* (WoT) (Caronni, 2000). In a WoT, users start with a set of people whom they trust and delegate as their trusted nodes. Users will trust people who are trusted by their trusted nodes. Overtime, a user's WoT can grow into many indirect trusts. One of the largest applications of WoT is a public blockchain. While a blockchain node can operate trustlessly, any user must first decide which software of the blockchain they want to run. How can a user know that the *Bitcoin* software that they downloaded is not a malicious version? While they can check the hash of the software, how can they know the correct hash? Before they have the data to verify against, they must first rely on known and trusted users who vouch for the validity of the software. Similarly, large scale open-source projects such as *Debian*, an operating system, relies on trusted community members to avoid malicious code in the system (Wolf and Quiroga, 2018). I emphasize again that the main model does not assume any trust system between the lender and the borrower. Including a trust system only strengthens the result.

4.4 Adoption from other services

Adoption of credit rating scores play a major role in their value. For example, *FICO* scores are used by many lenders to evaluate a borrower's creditworthiness. The adoption of *FICO* gives its value since borrowers know that lenders will look at their scores. In my model, I study only the interactions between lenders and borrowers and do not assume the adoption of an anonymous undercollateralized loans history as a signal for creditworthiness. Given that borrowers can choose to default in anonymous markets, not defaulting should give a very strong signal that they are willing to honor their debts. Thus, if such an anonymous undercollateralized loans market is implemented in practice, it is likely that other lending services would want to look at borrowers' anonymous loans history. If this were to happen, it should give borrowers a greater incentive to be honest and build a good history of paying back undercollateralized loans.

One reason why I do not consider the benefits of honesty from external lenders is the concern that borrowers may try to fake their undercollateralized loans by lending to themselves to build credit history. This practice is applied not only in DeFi but also in traditional finance,[8] albeit it is more costly in the latter. Several countermeasures can help detect and deter borrowers from such a practice. Blockchain tracking can map relationships among accounts and determine whether a loan is genuine or the borrower's own loan. Nevertheless, it is important to note that lenders can solve this concern by considering only loans approved by themselves. The downside is that doing so can reduce the model's scalability as borrowers cannot use their credit history with other lenders to borrow from a new lender, meaning that borrowers will be restricted to a set of their frequent lenders.

4.5 Use of covenants

The main model does not assume the possibility of having covenants and allowing borrowers to use the funds however they wish. This assumption is conservative because allowing free use of funds opens the door to moral hazard for undercollateralized loans. However, as in traditional loans, DeFi loans conducted using smart contracts can have covenants, which restrict borrowers to only certain activities. The use of covenants in practice can help mitigate moral hazard because lenders can prevent borrowers from spending the funds on high risk activities. Furthermore, borrowers may signal trust to lenders by committing to spend the

[8] One such example is a service called *Self*

funds on only activities described in the loan application. Overall, it is likely that covenants can help instill trust and reduce moral hazard in an anonymous undercollateralized lending market.

5 Conclusion

In this paper, I show that it is possible to have anonymous undercollateralized loans, even under conservative assumptions. Such a market is achieved by setting the interest rate such that it is incentive-compatible for both lenders and borrowers, especially the borrowers to participate honestly. The model is free from identification issues because all relevant parameters are observable. What sets this paper apart from previous studies is the anonymity of market participants on undercollateralized loans, something that has been considered impossible.

As a direction for future research, although simulation results have confirmed the theoretical predictions, if this market is implemented in practice, it is possible that players are not aware of the long-term benefits of honesty and deviate from the optimal strategy. Research using primary data, especially real-life implementation, can help show whether an anonymous undercollateralized lending market is indeed possible. Given the conservative assumptions in this paper, it is possible that the maximum incentive-compatible interest rate can be higher in practice. A higher maximum interest rate means that the market can be both feasible and achieve market-clearing condition, i.e., supply = demand. A more efficient upper bound for incentive-compatible credit rate should prove useful for both academic and industrial uses.

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A Proof of Propostion 1.A and 1.B

A.1 Proof of Propostion 1.A

Proof. Let $N \in \mathbb{N}$ the number of periods that the borrower will be honest after getting access to undercollateralized loans ($N > \tau$). Further, for ease of notation, let $a = \mathrm{e}^{-i}$. After N periods, the borrower will default. To prove Proposition 1.A, it is sufficient to show that $\mathbb{E}[MF]$ can only be either always increasing in N or always decreasing in N, $\forall N \in \mathbb{N}$.

Starting from the first period that the borrower receives undercollateralized loans, the expected payoff from MF-strategy can be divided into three parts.

The first part is the gains from being honest for N periods, D_1:

$$D_1 = \sum_{t=0}^{\infty} \left(\sum_{j_1=t(N+\tau+1)+1}^{t(N+\tau+1)+N} a^{j_1} \right) H_B$$

The second part is the gains from all defaults, D_2:

$$D_2 = \sum_{t=0}^{\infty} \left(a^{((t+1)N+t\tau+t+1)} \right) S_B$$

The third part is the gains from all τ periods of overcollateralized loans, D_3:

$$D_3 = \sum_{t=0}^{\infty} \left(\sum_{j_3=t(N+\tau+1)+2}^{(t+1)(N+\tau+1)} a^{j_3} \right) B_B$$

Using Proof by Induction, it can be shown that:

$$\sum_{j=k}^{v} a^j = \frac{a^k - a^{v+1}}{1-a}, \quad \forall k, v \in \mathbb{N}, a \in \mathbb{R}, |a| < 1 \tag{8}$$

Thus, D_1, D_2, and D_3 can be expressed as:

$$D_1 = \sum_{t=0}^{\infty} \left(a^{(t(N+\tau+1)+1)} - a^{(t(N+\tau+1)+N+1)}\right) \frac{1}{1-a} H_B = \frac{a \cdot \left(1-a^N\right)}{1-a^{N+\tau+1}} \cdot \left(\frac{1}{1-a}\right) H_B \tag{9}$$

$$D_2 = \sum_{t=0}^{\infty} \left(a^{((t+1)N+t\tau+t+1)}\right) S_B = \frac{a^{N+1}}{1-a^{N+\tau+1}} S_B \tag{10}$$

$$D_3 = \sum_{t=0}^{\infty} \left(a^{t(N+\tau+1)+2} - a^{(t+1)(N+\tau+1)+1} a^{j_3}\right) \frac{1}{1-a} B_B = \frac{a^{N+2} \cdot (1-a^\tau)}{1-a^{N+\tau+1}} \cdot \left(\frac{1}{1-a}\right) B_B \tag{11}$$

The second equal sign in each of the above equations can be proven using Proof by Induction. Hence, the expected payoff for MF-strategy, given any value of N is:

$$\mathbb{E}[MF] = \mathbb{E}[D_1+D_2+D_3] = \mathbb{E}\left[\frac{-B_B a^{N+2+\tau} + (-H_B + S_B)\, a^{N+1} + (B_B - S_B)\, a^{N+2} + H_B a}{(-1+a)\left(a^{N+\tau+1}-1\right)}\right] \tag{12}$$

Differentiate 12 w.r.t N yields:

$$\frac{\mathrm{d}\mathbb{E}[MF]}{\mathrm{d}N} = \frac{\ln(a)\left((B_B - H_B)\, a^{N+2+\tau} + (H_B - S_B)\, a^{N+1} - (B_B - S_B)\, a^{N+2}\right)}{(-1+a)\left(a^{N+\tau+1}-1\right)^2} \tag{13}$$

Observe that $\dfrac{\mathrm{d}\mathbb{E}[MF]}{\mathrm{d}N} \neq 0$ because:

$$(B_B - H_B)\, a^{N+2+\tau} + (H_B - S_B)\, a^{N+1} - (B_B - S_B)\, a^{N+2} \neq 0, \quad \forall N \in \mathbb{N} \tag{14}$$

Thus, $\mathbb{E}[MF]$ does not have a local optimum. It follows that $\mathbb{E}[MF]$ either always increasing or always decreasing in $N \in \mathbb{N}$. □

A.2 Proof of Proposition 1.B

Proof. Each period that M-type player defaults, he has the same payoff as S-type player, who defaults every period. Thus, to compare the expected payoff between M-type and S-type, it is sufficient to compare when M-type player chooses to repay. Because M-type player only repays if $H_B > S_B$, it follows that in every period that M-type player repays, his payoff is strictly higher than S-type player. In the periods following M-type's payback and S-type's default, M-type can choose to default while S-type must payback his overcollateralized loans. Since the option to default yields a loan cost of 0, and $c > 0$, M-type player receives strictly higher payoff in all τ periods that S-type player must do overcollateralized loans. It follows that M-type player's expected payoff is higher than S-type's expected payoff in all periods. □

B Proof of Proposition 2

Proof. The considered infinitely repeated game consists of infinite extensive form subgame. Let a subgame be a binary tree that starts with decision node and ends with decision nodes in all possible paths. By the assumption that return ρ_t is independently and identically distributed (i.i.d), it follows that all subgames are identical. Hence, to ensure that $\mathbb{E}[H(r) - M(r) > 0]$, it is sufficient to ensure that the expected differences in payoff between the always honest strategy and all other possible combinations are positive in a subgame.

For an M-type player in a subgame, he can have pure honest path, pure selfish path, and many paths with mixed decisions. I claim that in any identical subgame, the pure honest path and the pure selfish path last up to $\tau + 1$ periods; the strategy with mixed decisions lasts for $2\tau + 1$ periods. To see this, let $t = 0$ for any starting decision node. Because the borrower decides whether to default based on ex-post return, if he defaults, he will receive payoff S at $t = 1$. After τ periods of receiving B, he will be in another decision node. Thus, it takes $\tau+1$ periods for a borrower who defaulted at $t = 1$ to reach another decision node. If the borrower did not default for the first τ periods, but then defaulted at period $(\tau + 1)$-th, he will need another τ periods to reach a decision node. If he defaulted any time earlier than period τ-th, he must be able reach a decision node earlier. Hence, to ensure that a subgame is identical with all terminal node being decision node, one needs to consider only up to $2\tau + 1$ periods.

Consider an identical subgame with i.i.d collateral asset returns, the probability that an M-type player ends up in the pure honest path is $(1-F)^{\tau+1}$. If the returns are such that the M-type borrower chooses the pure honest path, his payoff is the same as that of the H-type borrower. Thus, the expected payoff difference for the honest branch is 0.

For the pure selfish path, since I only need to consider one default, the probability of this path is F, which occurs at the period after the starting node. Because M-type borrower only defaults if $\rho_t < r$, the expected payoff for when he defaults is conditional on $\rho_t < r$. The expected difference in payoff between M-type and H-type when M-type defaults is:

$$D_{HS} = \mathbb{E}\left[P\left(l\rho_t + l - r - 1\right) \mid \rho_t < r\right]$$

The expected payoff difference for the selfish branch is then:

$$HS(r) = \left(\mathrm{D}_{HS} \cdot \mathrm{e}^{-i} + \sum_{n=1}^{\tau} \left(\mathrm{D}_{HB} \cdot \mathrm{e}^{-i\cdot(n+1)}\right)\right) \cdot F \tag{15}$$

Finally, if the borrower chose to be honest initially, he can then switch to selfish at any point. Let n be the number of periods before the borrower switch to selfish. The probability for the borrower to be in any of the mixed paths is $(1-F)^n F$, i.e., honest for the first n periods then default. Because the borrower always decides ex-post, $n \geq 1$. In addition, since the borrower can default at any time before τ-th period, for the last τ periods are for overcollateralized loans, $n \leq \tau$. The expected payoff difference for any of the τ mixed paths can be described as:

$$HM(r) = \sum_{n=1}^{\tau} \left((1-F)^n \cdot F \cdot \left(\left(\sum_{k=1}^{\tau} \left(\mathrm{D}_{HB} \cdot \mathrm{e}^{-(k+n+1)\cdot i}\right) \right) + \sum_{j=1}^{n} \left(\mathrm{D}_{HH} \cdot \mathrm{e}^{-i\cdot j}\right) + \mathrm{D}_{HS} \cdot \mathrm{e}^{-i\cdot(n+1)} \right) \right) \tag{16}$$

Using the result from 8, $HS(r)$ and $HM(r)$ can be written as:

$$HS(r) = \frac{\left(-\mathrm{e}^{-i(\tau+1)}\mathrm{D}_{HB} + (\mathrm{D}_{HB} - \mathrm{D}_{HS})\,\mathrm{e}^{-i} + \mathrm{D}_{HS}\right) F}{\mathrm{e}^{i} - 1} \tag{17}$$

$$\begin{aligned} HM(r) = &\left(\frac{\mathrm{e}^{i}\,(1-F)^{\tau+1}\left(\mathrm{e}^{i}\mathrm{D}_{HB}\mathrm{e}^{-(2T+3)i} - \mathrm{e}^{i}\mathrm{D}_{HB}\mathrm{e}^{-(T+3)i} - \mathrm{D}_{HS}\mathrm{e}^{-i(2+T)}\mathrm{e}^{i} + \mathrm{D}_{HS}\mathrm{e}^{-i(2+T)}\right)}{(F+\mathrm{e}^{i}-1)\,(\mathrm{e}^{i}-1)}\right) F \\ &- \left(\frac{\mathrm{e}^{i}\,(1-F)\left(\mathrm{e}^{i}\mathrm{D}_{HB}\mathrm{e}^{-(T+3)i} - \mathrm{e}^{i}\mathrm{D}_{HB}\mathrm{e}^{-3i} - \mathrm{D}_{HS}\mathrm{e}^{-2i}\mathrm{e}^{i} + \mathrm{D}_{HS}\mathrm{e}^{-2i}\right)}{(F+\mathrm{e}^{i}-1)\,(\mathrm{e}^{i}-1)}\right) F \end{aligned} \tag{18}$$

where

$$\begin{aligned} \mathrm{D}_{HS} &= \mathbb{E}\left[H_B - S_B \mid l(1+\rho_t) < 1+r\right] \\ &= \mathbb{E}\left[P(l(1+\rho_t) - (1+r)) \mid l(1+\rho_t) < 1+r\right] < 0 \\ \mathrm{D}_{HB} &= \mathbb{E}\left[H_B - B_B\right] \\ &= P\left(c - r + l\bar{\rho}\right) + (k-l)\,Py\,(1+\bar{\rho}) \end{aligned}$$

Taking the sum $HS(r) + HM(r)$ yields the desired result.

$\square$

Table 1. Parameter definitions

Parameter	Meaning
P	loan amount
r	anonymous credit borrowing rate
τ	number of periods (years) before credit loans are allowed
i	discount or risk-free rate
c	overcollateral borrowing rate
u	unobservable borrower's return from using the loan
y	return the borrower gains from holding his collateral assets
k	collateral-to-loan ratio for overcollateralized loans ($k > 1$)
l	collateral-to-loan ratio for undercollateralized loans ($l \leq 1$)
ρ_t	stochastic collateral assets' return
F	the probability $Pr(\rho_t < r)$

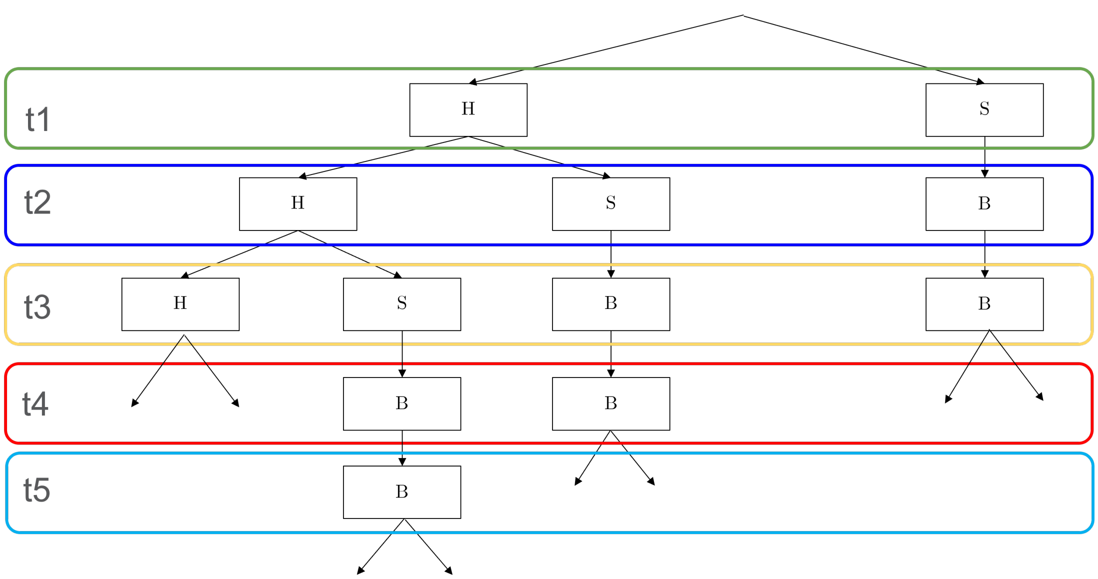

Figure 1. Visualization of possible actions by M-type borrower for $\tau = 2$. Each split (double arrows) represents a decision node, which indicates that the borrower gets to decide whether he will be honest, and receive payoff H, or selfish, and receive payoff S.

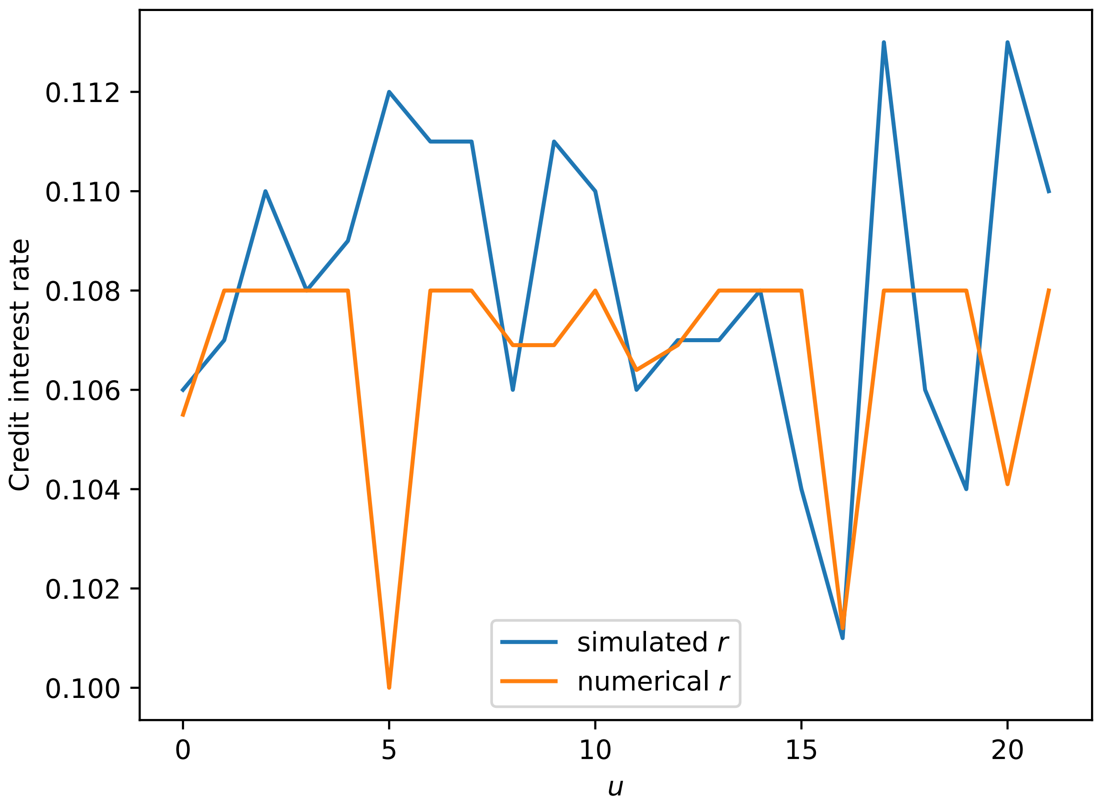

Figure 2. Simulated and numerical interest rate r. The simulated r is derived from simulation while the numerical r is found by solving $\mathbb{E}[H(r) - M(r)] = 0$ using Proposition 2.

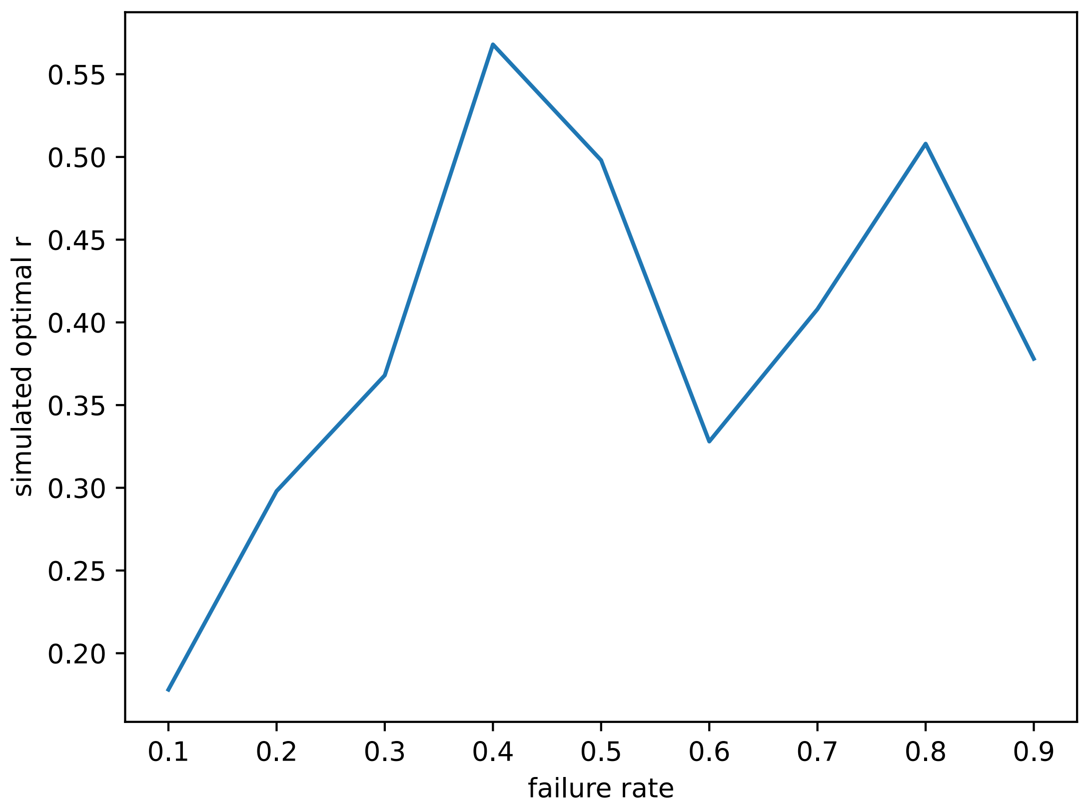

Figure 3. Simulated maximum $\bar{r}$ across different failure rate.

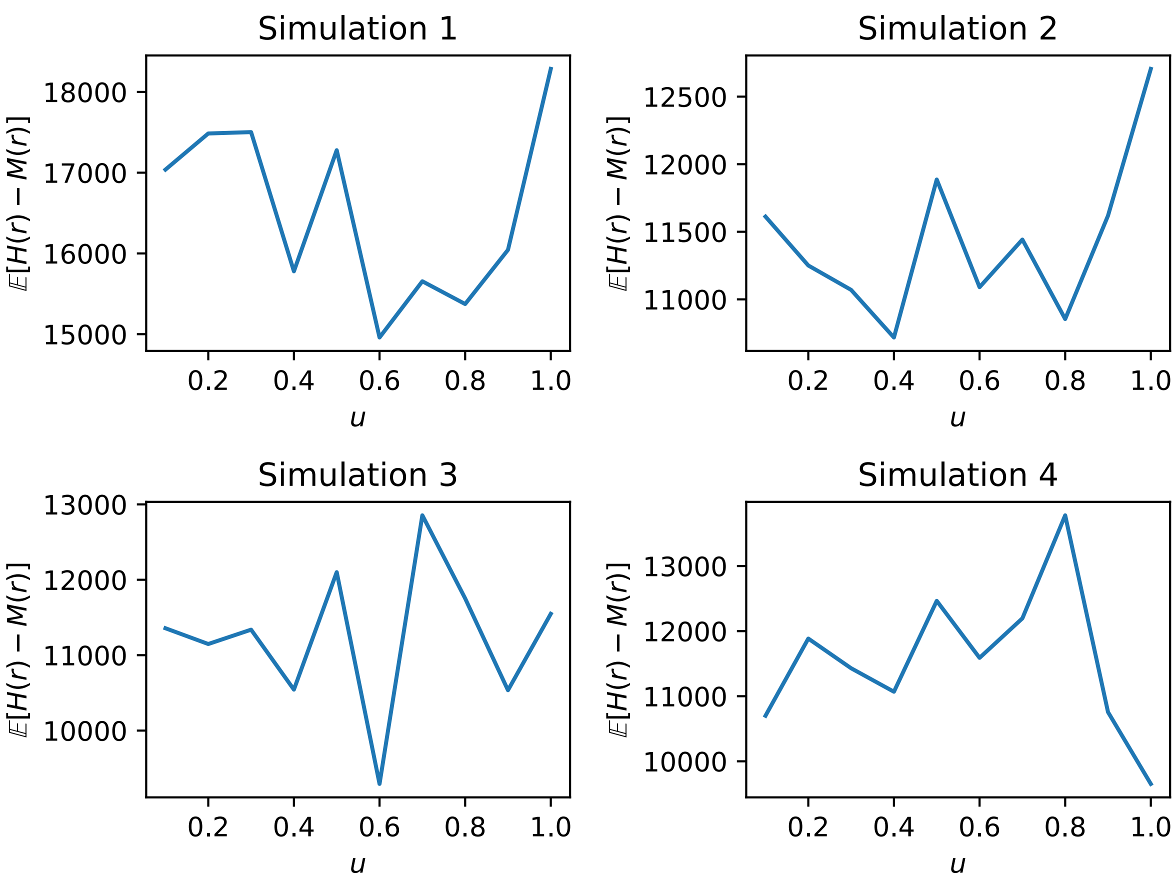

Figure 4. Simulated $\mathbb{E}[H(r) - M(r)]$ across u in the presence of moral hazard.

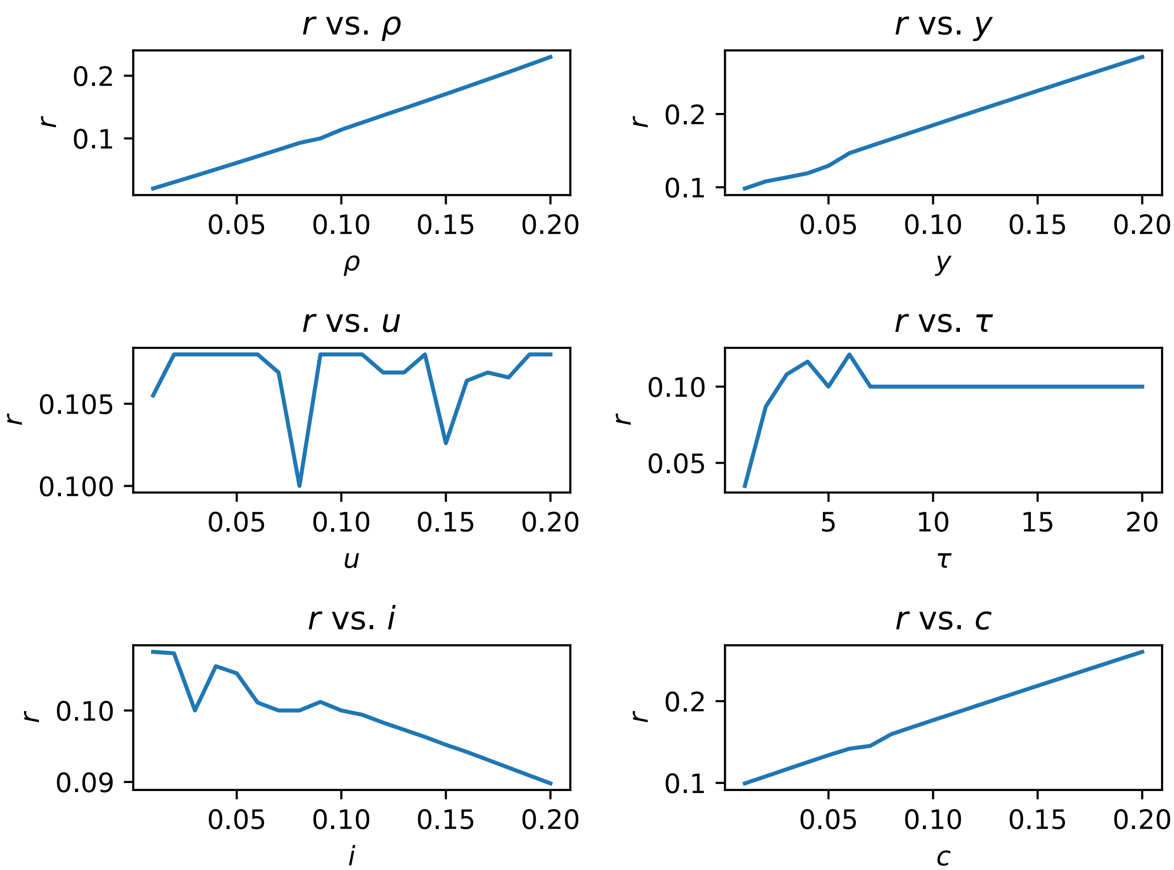

Figure 5. This figure plots the maximum credit interest rate r against key model parameters. The interest rate r is estimated numerically by solving for the root of 5. In each plot, except for the focal parameter, all other parameters are kept constant and as defined in 3.1.